SFEM using a volumetric-deviatoric split of the elasticity tensor

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In many engineering applications the materials are heterogeneous. Typical examples are adhesives, polycristallines and composites. This heterogeneity leads often to uncertainty in the material parameters and to uncertainty in the mechanical response. Therefore, macroscopically heterogeneous materials should be modelled by a stochastic approach instead of a deterministic approach. Mathematically, the system can be described by stochastic partial differential equations (SPDEs) with stochastic fields, which are solved by the stochastic finite element method (SFEM). Mostly used methods in SFEMs are *Monte Carlo* (MC), *Galerkin* and *polynomial chaos expansion* (PCE) [1, 2].

The aim of this work is to consider the uncertainty of a linear elastic body. An adhesive material behavior is applied. The uncertainty is considered by random material parameters, which are modeled as stochastic fields. Therefore, unknown stochastic fields are expanded with the polynomial chaos method and a *Galerkin* approach is used to solve the associated unknown coefficients. The key idea of our contribution is the split of the linear elasticity tensor into a volumetric and deviatoric part. Then, from experimental data the distribution of the random variables, i.e. *Young's* modulus E and the *shear* modulus G, are known. Consequently, E and G are expanded with the PCE and a *Galerkin* projection [1] can be applied to reduce the SPDEs into a system of deterministic PDEs. Finally, the finite element matrix equation consists of three contributions, namely the deterministic, the variation of volumetric and the variation of deviatoric part.

As a numerical example we consider the static problem for uniaxial tension of the rectangular plate with a circular hole. This problem is investigated under plane strain conditions. Results of the deterministic solution and the influence of the distribution of the material parameters E and G on the volumetric and deviatoric solution are presented.

[1] H. G. Matthies and A. Keese. Galerkin methods for linear and nonlinear elliptic stochastic partial differential equations. *Comput. Methods Appl. Mech. Engrg.*, 194:1295–1331, 2005.

[2] R. G. Ghanem and P. D. Spanos. Stochastic Finite Elements: A Spectral Approach. Springer-Verlag, New York, 1991.