A TWO-STRAIN SPATIO-TEMPORAL MATHEMATICAL MODEL OF TUMOR GROWTH

Robert L. Álvarez[Δ], Fabio A. Milner* and John D. Nagy†

[Δ] Intel Corporation
5000 West Chandler Blvd, Chandler, AZ, 85226
robert.alvarez@intel.com

*Arizona State University
School of Mathematical and Statistical Sciences
P.O. Box 871804, Tempe, AZ 85287-1804
milner@asu.edu
† Scottsdale Community College
Department of Biology
9000 East Chaparral Road, Scottsdale, AZ 85256-2626
john.nagy@sccmail.maricopa.edu

ABSTRACT

Cancer in most organs develops in the form of solid tumors that begin a spheroids consisting entirely of cancerous (parenchyma) cells. When the tumor reaches the size of approximately 1 mm, it needs blood vessels (vascularization) to develop in it in order to be able to develop further. A model of vascularized tumor growth with competing parenchyma cells of two different strains will be presented. A related model for a single strain of parenchyma was described by Duan and Friedman in [1]. The model consists of a free-boundary value problem for a system of nonlinear parabolic PDEs describing the dynamics of the parenchyma cells and vascular endothelial cells (VECs), coupled with an ODE describing the dynamics of vascular density and an algebraic equation describing a variable used as proxy for resource availability locally. The tumor is assumed to have radial symmetry and the mathematical domain is then taken to be a sphere with moving boundary, with the boundary moving at the unique physically determined velocity that changes local tumor volume exactly to maintain density.

Some theoretical analytical properties of the model will be presented, and then the focus will shift to simulations to study the possible development of hypertumors [2]. Results from several simulations will be shown and biological interpretations provided, including how selection for increased proliferation or for increased angiogenesis may lead to tumors that are structurally like integrated tissues or like segregated ecological niches, as well as to hypertumors.

REFERENCES

[1] D. Chen and A. Friedman, "A two-phase free boundary problem with discontinuous velocity: Application to tumor model", J. Math. Anal Applic, **399** (1), 378-393 (1991).
[2] J. Nagy, "Competition and Natural Selection in a Mathematical Model of Cancer", *Bull. Math. Biol*, **66**, 663-687 (2004).