Solutions of Geomorphic Stefan Problems

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ABSTRACT

A moving boundary problem requires the solution of a field transport equation in which the location and movement of one or more of the domain boundaries are a-priori unknowns. The classic moving boundary problem involving the phase change of ice, referred to as the Stefan Problem, involves the tracking of the water/ice interface during melting. When we restrict attention to one-dimensional domains there is a rich diversity of closed form solutions for Stefan problems (see examples in [1-2]). All of these solutions provide excellent benchmarks for proving more general multi-dimensional numerical solutions. If, however, we were to restrict attention to closed solutions that have an immediate physical application over those that are obtained through mathematical construction alone, the number of solution available to us would be noticeably reduced. Recently a new class of Stefan like problems, related to the tracking of fronts in growing sedimentary systems such as ocean deltas and desert fans, has been a keen area of research interest (see examples in [3-4]). These problems exhibit physical features that allow for the development of models and closed form solutions that in other physical systems, such as heat transfer, would not be realistic. In this paper we explore recent and new closed form solutions of these so-called geomorphic Stefan problems. The first, related to the formation of an ocean delta, is analogous to a melting problem in which the latent heat (in this case the ocean depth) increases as a function of space. The second problem also involves the movement of an ocean shoreline but with a fixed angle (gradient) of sediment repose at the front; a feature that requires a modification of the standard Stefan condition. This later problem is then generalized to consider two sediment grain densities; a situation that requires tracking a second internal front in addition to the shoreline. The last problem is the building of an axisymmetric sediment desert fan in which the diffusivity increases as a function of the radial distance. This behavior allows us to bypass the singularity at the origin and develop a closed form solution that indicates a front movement as the cubed root of time; consistent with mass balance arguments.

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