Reduced Basis Method for Variational Inequalities in Contact Mechanics

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Abstract

We present an efficient model order reduction method [1] for parametrized elliptic variational inequalities of the first kind: find $u \in K$ such that:

$$a(u, v-u; \mu) \geq f(v-u; \mu), \quad \forall v \in K(\mu)$$

where $K(\mu) := \{v \in H^1(\Omega) | Bv \leq g(\mu)\}$.

Motivated by numerous engineering applications that involve contact between elastic body and rigid obstacle, e.g. the obstacle problem [2], we develop a primal-dual reduced basis approach to construct offline-online efficient yet certified reduced order models. Such models find application in the real-time or many query context of PDE-constrained optimization, control, or parameter estimation. They can also be easily extended to parabolic systems [3].

Firstly, we develop a primal-dual certified reduced basis method for bijective constraint operator B that provides sharp and inexpensive *a posteriori* error bounds. We compare both the proposed error bounds and the computational costs with the proposal in [4], demonstrating the quality and effectivity of the approximation and the error bounds. Then, we extend our results to more generalized problems, namely an injective constraint operator B, e.g. for a generalized obstacle problem, or a surjective constraint operator B, e.g. in Signorini's Problem. We present both *a priori* and *a posteriori* analysis for the generalized formulation [2] and the saddle point formulation [5]. Lastly, we discuss the construction and sampling procedure [1] for the given method.

References

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