Essential Boundary Conditions and k-Compressibility in Hyper Reduced Order Models

Alejandro Cosimo[1,*], Alberto Cardona[1] and Sergio Idelsohn[1,2]

[1] Centro de Investigación de Métodos Computacionales (CIMEC)
Santa Fe, Argentina

[2] International Center for Numerical Methods in Engineering (CIMNE) and Institució Catalana de Recerca i Estudis Avançats (ICREA), Barcelona, Spain

ABSTRACT

The use of Hyper Reduced Order Models (HROMs) to tackle the high computational complexity of problems found in science and in engineering is an interesting option for real-time computations. These methods look for the solution in subspaces of the solution space with very small dimension, by making the hypothesis that the quantities to be hyper-reduced are *k-compressible* in a certain basis in the sense that these quantities have at most *k* non-zero significant entries when expressed in terms of that basis [1]. In HROMs based on the Proper Orthogonal Decomposition (POD), a new basis **X** is built by computing the POD of a set of snapshots that are time instances of the spatial distribution of the solution to a training problem. This basis inherits the behaviour of the elements of the snapshot's set, hindering the possibility of reproducing non-admissible test functions. That is why, careful attention must be paid on how the snapshots for building **X** are collected.

In this regard, treatment of inhomogeneous essential boundary conditions must be specially considered. Gunzburger et al. [2] analysed boundary conditions in the context of Reduced Order Models, and Carlberg et al. [3] gave a discussion about snapshot collection procedures for nonlinear problems. Here, we study this issue in detail in the context of HROMs. Besides, we examine different strategies for addressing hyper-reduction of problems characterised by multiple non-linear contributions. In order to improve performance and robustness, we propose to separately hyper-reduce each non-linear term.

Applications to nonlinear phase change problems with temperature dependent thermophysical properties and time dependent essential boundary conditions are considered. However, the techniques developed can be applied in other fields as well. Through these examples we show the improvements in performance and robustness of the introduced HROM.

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