Goal-Oriented Adaptivity using Unconventional Error Representations

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ABSTRACT

In the scope of subsurface modelling via the resolution of inverse problems, the so-called goal-oriented adaptivity plays a fundamental role. Indeed, while classical adaptive algorithms were first designed to accurately approximate the energy norm of a problem, one requires a good approximation of a specific quantity of interest. An energy norm driven self-adaptive strategy can still be used for that purpose, although it often becomes sub-optimal and unable to provide an accurate solution for the required quantity of interest in a reasonable amount of time.

During the late 90's, to overcome this issue, the so-called goal-oriented strategy appeared. The goal-oriented approach consists in expressing the error in the quantity of interest as an integral over the entire computational domain involving the errors of the original and adjoint problems, and then minimise an upper bound of such error representation by performing local refinements. Most authors, using the adjoint problem, represent the approximation error in the quantity of interest via the global bilinear form that describes the problem in terms of local and computable quantities.

Our methodology, however, is based on the selection of an alternative bilinear form exhibiting better properties than the original bilinear form (e.g. positive definiteness). We represent the residual error functional of the adjoint problem through this alternative form. We can then compute new upper bounds of the error of the quantity of interest in a similar way than with the classical approach.

Our main contribution is to demonstrate that a proper choice of such alternative form may improve the upper bounds of the error representation. Moreover, the method proposed here generalises the existing ones, since, in particular, we can select as the alternative bilinear form the one associated to the adjoint problem. The proposed method can be applied to rather general 1D, 2D and 3D problems. We describe and illustrate it numerically with a 1D-Helmholtz example. As we shall illustrate, the advantages of the proposed method are clear from the simplest 1D problem. Our upper bounds are sharper than the classical ones if one selects wisely the alternative operator. Extensive numerical results are illustrated using uniform h- and p-refinements, as well as a simple self-adaptive goal-oriented p-refinement strategy.

The application to other adaptive algorithms such as a goal-oriented hp-adaptive algorithm, adaptivity in a high continuity space, or adaptivity in time domain is straightforward.