ECSW: An Energy-based Structure-preserving Method for the Hyper Reduction of Nonlinear Finite Element Reduced-Order Models

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ABSTRACT

The computational efficiency of a typical projection-based nonlinear model reduction method hinges on the efficient approximation, for explicit computations, of the projection onto a subspace of a residual vector. For implicit computations, it also hinges on the additional efficient approximation of the projection of the Jacobian of this residual with respect to the solution. The Empirical Interpolation Method (EIM) [1], its discrete counterpart DEIM [2], the Gauss-Newton with Approximated Tensors (GNAT) method [3,4], and the Gappy Proper Orthogonal Decomposition method (Gappy POD) [5] are popular methods for performing such approximations. They differ in several aspects such as their applicability at the continuous, semi-discrete, or fully discrete level(s), their underlying left and right projectors, and their suitability for explicit and/or implicit schemes. However, they all share accuracy as the primary driver for their algorithmic design. They have also all demonstrated various forms of success for the reduction of nonlinear computational models emanating from elliptic, parabolic, and first-order hyperbolic partial differential equations. The first objective of this talk however is to show that they all lack robustness for second-order nonlinear dynamical systems, because they do not necessarily preserve the numerical stability properties of the discrete computational models they reduce. Consequently, the second objective of this talk is to present ECSW [6], an Energy-Conserving Sampling and Weighting method for the model reduction of nonlinear second-order dynamical systems such as those arising, for example, in structural dynamics, solid mechanics, wave propagation, and device analysis. This proposed hyper reduction method is physics-based and natural for finite element semi-discretizations. It is applicable at both the semi-discrete and discrete levels. Unlike all aforementioned reduction methods, it preserves the Lagrangian structure associated with Hamilton's principle [7], and therefore preserves the numerical stability properties of the nonlinear discrete system it reduces. It will also be shown that the error committed by ECSW during an online approximation is bounded by the error committed during the offline approximation of the training samples. Therefore, this online error can be estimated *a priori* and is controllable. The performance of ECSW will be first demonstrated for a set of academic but nevertheless challenging nonlinear dynamic response problems taken from the literature, and compared to that of DEIM and its unassembled variant recently introduced for finite element computations under the name UDEIM [8]. Next, the potential of ECSW for complex second-order dynamical systems with strong nonlinearities will be highlighted with the realistic simulation of the transient response of a generic V-hull vehicle to an underbody blast event. For this hihgly nonlinear time-dependent problem, ECSW will be shown to deliver an excellent level of accuracy while enabling the reduction of CPU time by more than *four orders of magnitude*.

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