

# Advances in Isogeometric Analyses – (a) Contact, (b) Beams and Shells

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## ABSTRACT

The higher inter-element continuity of NURBS functions is a key property of the Isogeometric Analysis (IGA). The present contribution describes two subjects where this feature is exploited, namely in the context of contact formulations and for thin-walled structures like beams, plates and shells.

The applied *IGA contact formulation* utilizes a Lagrange Multiplier Method avoiding numerical instabilities often present in a Node-To-Segment (NTS) approach. The present Point-To-Segment (PTS) formulation places collocation points directly on the contact surface [1]. However the patch test is still not satisfied. This problem can be remedied if the contact integral is not collocated but numerically integrated applying the Segment-To-Segment (STS) contact formulation [2]. In this case the quadrature points play the role of evaluation points of the contact equation. PTS and STS differ only in the underlying Lagrange multiplier field, in number, location and weights of these points, as well as in their efficiency.

In the present study the PTS formulation is equipped with features of the STS version, namely also introducing a field for the Lagrange multiplier, but still collocating the contact integral, for example at Greville or Botella points. Weights are assigned to the expressions at these points reflecting the corresponding influence regions. Numerical examples show the same accuracy as the original PTS algorithm; they represent directly the contact stresses instead of contact forces, but are more efficient than the STS approach.

The second part of this contribution addresses a *hierarchical formulation for beams and shells*. An important advantage of IGA is the straightforward implementation of classical theories, like the Euler-Bernoulli beam or Kirchhoff-Love shell formulations, requiring  $C^1$ -continuity. Based on these elementary models shear deformable theories, like Timoshenko beam and Reissner-Mindlin shell formulations, can be derived in an elegant way applying a hierarchical concept as a pure displacement model within IGA [3]. In contrast to the usual parameterization with midpoint displacements and total rotations of the director, the transverse shear angles are introduced representing incremental rotations as primary variables. Consequently transverse shear and curvature-thickness locking (in case of 3D-shells) are removed per definition. Alternatively the displacement field can be decomposed into a bending part and shear parts avoiding rotational parameters and substantially improving the stress resultant quality [4].

Current investigations concentrate on corresponding schemes avoiding membrane locking.

## REFERENCES

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