Remarks on modeling of static friction in closed-loop kinematic chains

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Abstract
It has been shown that when a multibody system is overconstrained and Coulomb-like joint friction is present (i.e., friction that depends on normal reactions in joints), the simulated motion of the system is not unique [1]. In this study mechanisms without redundant constraints are considered. The investigation is focused on the stiction phase and its vicinity, when the relative joint velocity is equal or close to zero. It is shown that problems with uniqueness of solution may be experienced in these circumstances. Three different models of friction are investigated.
Consider a multibody system described by a *n*-element vector of dependent coordinates **q** that are subject to *m* holonomic constraints. It is assumed that constraints are independent, i.e., no redundant constraints exist. The considered system forms a closed-loop kinematic chain. For the sake of simplicity, it is assumed that the system has exactly one degree of freedom ($m = n–1$).
At a certain moment of the system motion, the relative velocity in joints can reach a value of zero. After that event, motion of the system may continue instantaneously or, when stiction occurs, can be stopped for some nonzero period of time. According to the Coulomb model, no relative joint motion is observed (the joint stays locked) as long as the joint friction force remains within an allowable range. This condition for i^{th} joint can be written as follows [2]:

$$\left\|\mathbf{F}_i\right\| \le \left\|\mu_i^S \cdot \mathbf{N}_i\left(\mathbf{q}, \mathbf{C}_i\right)\right\|, \tag{1}$$

where $\mathbf{F}_i$ is a physical friction force, μ_i^S is a coefficient of static friction and $\mathbf{N}_i$ is a physical normal force that depends on generalized constraint reactions $\mathbf{C}_i$ as well as on the joint geometry and on its instantaneous configuration (expressed in terms of coordinates **q**).
In multibody modeling, the joint locking due to static friction can be represented by an additional constraint imposed on the system [2] (the constraint is activated when the joint relative velocity is numerically close to zero). The reaction associated with this constraint corresponds to the joint static friction force. Condition (1) is repeatedly checked during simulation, to determine whether stiction occurs. If condition (1) is not satisfied, the additional constraint is deactivated and the joint transits from stiction to sliding.
In our case of a 1-DOF mechanism, the relative velocities in all kinematic pairs become equal to zero at the same moment. Thus, additional constraints must be simultaneously added to all joints with friction. Since the number of additional constraints *k* exceeds the number of degrees of freedom (by $k–1$), the multibody system becomes redundantly constrained. As a consequence, neither normal reactions nor static friction forces can be uniquely determined. Moreover, in this study we show that for many cases it is impossible to decide whether or not stiction conditions (1) are fulfilled for given external loads: two substantially different solutions – one corresponding to sliding and the other corresponding to stiction – can be found.
In practical applications, the constraint addition-deletion approach is seldom used to deal with friction "discontinuity" at zero relative joint velocity. More frequently employed models treat both stiction and sliding regimes uniformly, i.e. the same friction model is used in both regimes and no additional constraints are imposed. In these models, friction force is described by equality rather than inequality and strongly depends on velocity of sliding in the vicinity of stiction. For example, the simplified Coulomb model is obtained by "smoothing" the friction law in the region of zero slip speed [3]:

$$\mathbf{F}_i = \mu_i^K \tanh\left(k_i\, v_i\right) \cdot \mathbf{N}_i\left(\mathbf{q}, \mathbf{C}_i\right), \tag{2}$$

where v_i is the relative joint velocity, k_i is a large constant, and μ_i^K is a coefficient of kinetic friction.
The LuGre model [4] is more advanced and closer related with friction phenomena at microscopic level. This model includes the "internal dynamics" state variable z_i that represents the average deflection of asperities at contacting surfaces. The joint friction force is calculated as:

$$\mathbf{F}_i = \left(\sigma_i^0 z_i + \sigma_i^1 \dot{z}_i\right) \cdot \mathbf{N}_i\left(\mathbf{q}, \mathbf{C}_i\right), \quad \dot{z}_i = v_i - z_i\, \sigma_i^0 \left|v_i\right| / G\left(v_i\right), \quad G\left(v_i\right) = \mu_i^K + \left(\mu_i^S - \mu_i^K\right) e^{-(v_i / v_i^S)^2}, \tag{3}$$

where σ_i^0, σ_i^1, and v_i^S are constant coefficients.
All discussed here models of friction describe the same physical phenomena; hence similar results of using them should be expected. It is reasonable to ask, how the problems with solution uniqueness, encountered when the constraint addition-deletion method is used, are reflected in the other models.
In our study a simple planar 1-DOF mechanism (Fig. 1a) was simulated and three different friction models were investigated. In all simulations the initial conditions and the time histories of external loads (**P**, $\mathbf{P}_1$, $\mathbf{P}_2$) were the same. Moreover, coefficients of friction were recognized as primary factors characterizing frictional behavior of the system and were kept unchanged during all simulations.
In the case of constraint addition-deletion approach combined with the Coulomb model, various methods of handling redundant constraints were applied. Since the way of dealing with redundancy affects obtained values of reactions, distinctly different friction forces were found in tested variants.
In the case of the simplified Coulomb model, simulations were conducted for diverse values of k_i parameters. It was observed that the mechanism behavior, e.g., friction forces and the moment of transition from rest to motion, strongly depends on the chosen values.
In the case of LuGre model, simulations were conducted for various values of parameters other than coefficients of friction. The choice of parameters' values was essential for results calculated in the stiction phase. As an example, friction forces obtained for diverse values of σ_1^0 are presented in Fig 1b.

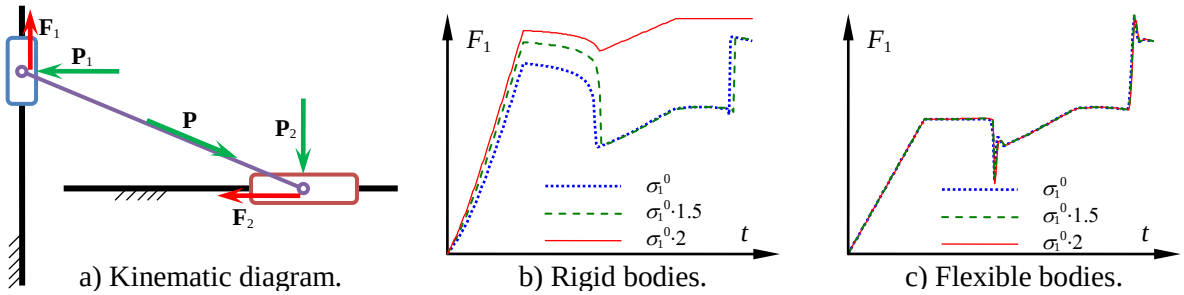

Figure 1: A 1-DOF mechanism and LuGre friction forces calculated for diverse values of σ_1^0.

Flexibility of bodies is one of the main factors deciding on the distribution of reactions in overconstrained mechanisms. It may be expected that flexibility strongly influences (or even determines) reactions in the stiction phase as well. Thus, the multibody model was adjusted to account for elasticity of bodies and the discussed earlier simulations were repeated. The results reveal that this time the flexibility of bodies is the key factor influencing calculated reaction and friction forces. The selection of values of friction model parameters (except for coefficients of friction) is far less relevant. Exemplary results of LuGre friction forces obtained for various values of σ_1^0 are presented in Fig 1c.
It this study we show that in the case of rigid body mechanism, in the stiction regime, friction model parameters of secondary importance (whose values are often quite arbitrarily chosen, predominantly to meet some numerical requirements), e.g., k_i or σ_i^0, strongly influence calculated reactions and friction forces. Consequently, the moment of stiction to sliding transition is affected by the choice of values of these parameters. On the other hand, when flexibility of bodies is taken into account, the selection of values of "secondary" parameters only negligibly influences the results of simulations.
The distribution of joint reactions is crucial for friction forces modeling, therefore results of stiction phase simulations, obtained using purely rigid body models, may be considered as doubtful.

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References

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