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Symmetry-breaking due to Coulomb friction

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Abstract

There are several mechanical systems in which the occurrence of dry (Coulomb) friction breaks the symmetry in the structure of the corresponding equations of motion. For example, if the normal force between two bodies depends on the acceleration of the centroids, the mass matrix may become asymmetric. This asymmetry may even lead to the loss of stability of the desired motion. The incorporation of such effects in linear models of brakes was extensively studied, see e.g., [1].

The mechanical model considered in the present paper does not show this type of asymmetry in the equations of motion. However, pairs of asymmetric periodic solutions were found, with different maximal displacements in the positive and negative directions measured from the equilibrium.

The simplest model where the existence of asymmetric solutions was pointed out was a SDoF harmonically excited linear oscillator with dry friction, with equal friction coefficients of sticking S_1 and sliding S [2]. The dimensionless equation of motion assumes the form

$$\ddot{x}+x=\cos\left(\Omega\left(t+t_0\right)\right)-Sf\left(\dot{x}\right), \quad \text{where} \tag{1}$$

$$f\left(\dot{x}\right)\in\begin{cases}1 & \text{if } \dot{x}>0\\ \left[-S_1/S, S_1/S\right] & \text{if } \dot{x}=0\\ -1 & \text{if } \dot{x}<0\end{cases}$$

is a set-valued function. The symmetry-breaking solutions were derived analytically for the case of equal coefficients of friction ($S_1 = S$) and it was shown that they exist only at certain sub-harmonic resonant frequencies $\Omega = 1/2,\ 1/4,\ 1/6$, etc. Later it was found that the frequency domain of asymmetric solutions opens up if the ratio of sticking and sliding coefficients of friction is large, see Figure 1.

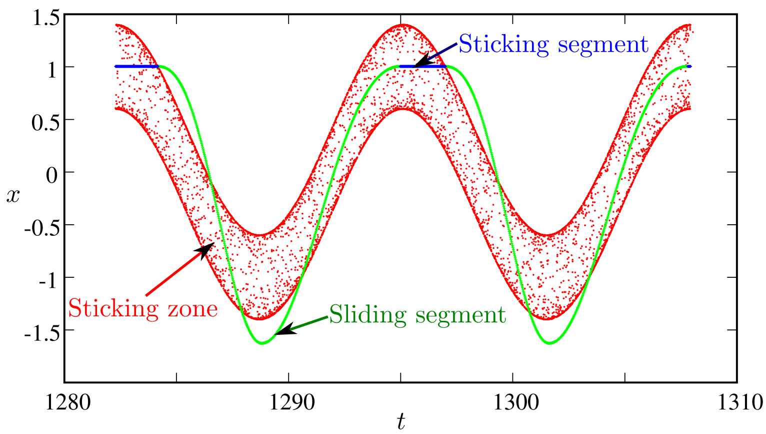

Figure 1: An asymmetric periodic solution at $\Omega = 0.49$, $S = 0.2$ and $S_1 = 0.4$.

The special non-smooth bifurcations of so-called Filippov systems have already been classified and several methods were developed for their analysis [3]. However, the analytical treatment of the case $S_1 \neq S$ is rather difficult, because the resulting non-smooth equation of motion does not belong to the group of

Filippov systems. In the examined case the vector field corresponding to sticking cannot be expressed as a convex combination of the vector fields that govern the sliding with positive or negative velocities. Another field of mathematics that deals with dry friction systems uses the notion of maximal monotone operators [4] – but the difference of sticking and sliding coefficient of friction means that the corresponding operator is not maximal monotone.

This is why we opted for a numerical search for special solutions. We used a brute-force approach and a continuation method, too. The obtained bifurcation diagram, shown in Figure 2, revealed the existence of chaotic solutions that are related to the symmetry-breaking. The parameter domains of periodic,

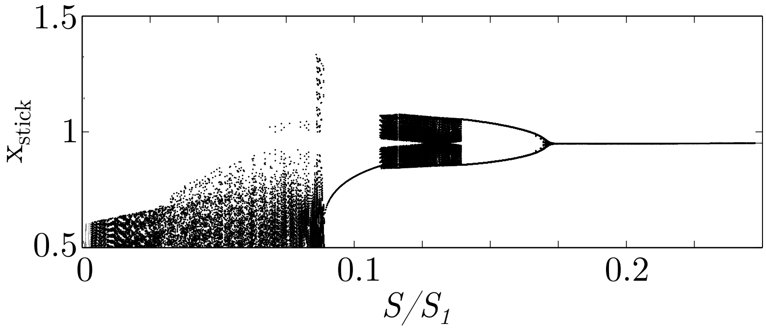

Figure 2: The positions of sticking segments versus the ratio S/S_1. $\Omega = 0.5$, $S_1 = 0.4$.

chaotic and transient chaotic solutions were explored and the Lyapunov exponent of the system was also calculated at several parameter values [5].

The aforementioned symmetry-breaking is related to a (non-smooth) crossing-sliding bifurcation. However, traces of symmetry-breaking behaviour were detected in a similar model with smooth LuGre friction, as well. Thus, we expect to encounter such special motions in real mechanical systems, too. The construction of a test rig is under way to proove the computational results. The different coefficients of friction will be achieved with the help of spherical roller thrust bearings where the starting torque and operational resistance torque are different.

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