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Exploiting the equations of motion for biped robot control with enhanced stability

Johannes Mayr, Alexander Reiter, Hubert Gattringer, Andreas Müller

Institute of Robotics
Johannes Kepler University
Altenbergerstr. 69, 4040 Linz, Austria
[joh.mayr, alexander.reiter, hubert.gattringer, a.mueller]@jku.at

Abstract

The scope of the present paper is the derivation of the equations of motion for humanoid robots, in particular legged robots. The derivation is performed in a modular and structured manner and it is shown how these equations can be exploited for the control of biped robots. The used methods allow to easily adopt the kinematic structure of single limbs and to reuse results obtained for limbs with similar kinematic structure but different inertial parameters such as in case the left leg is just a mirrored version of the right one. After finding a recursive formulation to calculate the equations of motion we perform various state transformations and apply some model simplifications to gain equations that can be used to effectively solve control problems.

First the equations of motion for a humanoid robot with floating base are derived. As the kinematic structure and the number of degrees of freedom may vary between different robots, it is convenient to introduce subsystems for various limbs. This allows easy exchange for example a complex arm with many degrees of freedom with a simpler one and vice versa. A subsystem consists of structural elements and actuators, which themselves can be grouped into smaller subsystems. Thus we start with the smallest subsystem consisting of a motor, a gear and a structural element attached to the driven side of the gear (subsequently referred to as drive-subsystem) as shown on the left side of Fig. 1 and compute the subsystem equations using the formulations in [1]. Next we adopt above subsystem with the inertial parameters of the individual drive-subsystems of the considered limb and then recursively combine these drive subsystems to the limb-subsystem (see Fig. 1 in the middle). Then the equations of motion of the complete robot are synthesized by recursively assembling the different limb-subsystems. The head and the upper-body of the robot are also treated as limb-subsystems. As a result we find the equations of motion of the humanoid robot system with floating base to

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{s}}+\mathbf{h}(\dot{\mathbf{s}},\mathbf{q})=\begin{pmatrix}0\\ \boldsymbol{\tau}_J\end{pmatrix}+\sum_j \kappa_j\begin{bmatrix}\mathbf{Ad}^T_{T_{jB}}\\ \mathbf{J}^T_{Bj}\end{bmatrix}\mathbf{W}_j. \tag{1}$$

where $\dot{\mathbf{s}}=\begin{pmatrix}\dot{\mathbf{r}}_B^T & \boldsymbol{\omega}_B^T & \dot{\mathbf{q}}_J^T\end{pmatrix}^T$ are the generalized velocites of the system, $\mathbf{M}$ and $\mathbf{h}$ denote the inertia matrix and nonlinear terms, $\boldsymbol{\tau}$ and $\mathbf{W}_j$ are the acuator torques and the contact wrenches at the feet.

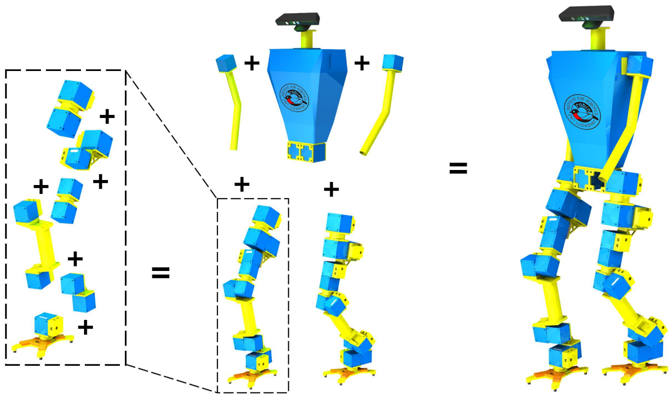

Figure 1: Subsystems are recursively combined to larger subsystems and finally to the whole robot.

By introducing a contact model we can now use Eq. (1) to simulate the dynamic behavior of a biped robot to evaluate the stabilization algorithms presented later.

Generally different generalized coordinates can be used for $\dot{\mathbf{s}}$ in Eq. (1). Instead of prescribing the upper body twist, commonly the velocity of the center of mass $\dot{\mathbf{r}}_C$ is generated by walking pattern generators

[2]. Considering the principles of virtual work [1], we can transform Eq. (1) to the new generalized coordinates $\dot{\mathbf{s}} = \begin{pmatrix} \dot{\mathbf{r}}_C^T & \boldsymbol{\omega}_B^T & \dot{\mathbf{q}}_J^T \end{pmatrix}^T$

$$\begin{bmatrix} m\mathbf{I} & 0 & 0 \\ 0 & \mathbf{M}_{\omega_B}(\mathbf{q}_J) & \mathbf{M}_{\omega_B,J}(\mathbf{q}_J) \\ 0 & \mathbf{M}_{\omega_B,J}^T(\mathbf{q}_J) & \mathbf{M}_J(\mathbf{q}_J) \end{bmatrix} \ddot{\mathbf{s}} + \begin{pmatrix} -m\mathbf{g} \\ \mathbf{h}_{\omega_B}(\dot{\mathbf{s}},\mathbf{q}_J) \\ \mathbf{h}_J(\dot{\mathbf{s}},\mathbf{q}_J) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \boldsymbol{\tau}_J \end{pmatrix} + \sum_j \kappa_j \begin{bmatrix} \mathbf{I} & 0 \\ \tilde{\mathbf{r}}_{cj} & \mathbf{I} \\ & \mathbf{J}_{J,j}^T \end{bmatrix} \mathbf{W}_j. \tag{2}$$

The joint angles $\mathbf{q}_J$ can easily be stabilized by a simple high gain position control law. Using the singular perturbation theory in addition it is possible to reduce the dynamical model to

$$\begin{bmatrix} m\mathbf{I} & 0 \\ 0 & \mathbf{M}_{\omega_B} \end{bmatrix} \begin{pmatrix} \ddot{\mathbf{r}}_C \\ \dot{\boldsymbol{\omega}}_B \end{pmatrix} + \begin{pmatrix} -m\mathbf{g} \\ \mathbf{M}_{\omega_B,J}\ddot{\mathbf{q}}_J^d + \mathbf{h}_{\omega_B} \end{pmatrix} = \sum_j \kappa_j \begin{bmatrix} \mathbf{I} & 0 \\ \tilde{\mathbf{r}}_{Cj} & \mathbf{I} \end{bmatrix} \mathbf{W}_j. \tag{3}$$

While (3) still considers the full multi-body dynamics, perfect tracking of the joints is assumed.
Equation (3) can be rewritten in terms of canonical momenta as

$$\begin{pmatrix} \dot{\mathbf{P}} \\ \dot{\mathbf{L}} \end{pmatrix} + \begin{pmatrix} -m\mathbf{g} \\ 0 \end{pmatrix} = \sum_j \kappa_j \begin{bmatrix} \mathbf{I} & 0 \\ \tilde{\mathbf{r}}_{Cj} & \mathbf{I} \end{bmatrix} \mathbf{W}_j \tag{4}$$

where $\mathbf{L}$ is the overall angular momentum and $\mathbf{P}$ is the total linear momentum.
With Eq. (3) and Eq. (4) we found equations governing the robot dynamics, that can now be employed for different control purposes.
In the walking pattern generation typically reference trajectories for the lower body coordinates are designed according to the dynamic constraints of the unilateral contact of the feet to the ground. While in most cases only the overall linear momentum $\mathbf{P}$ from Eq. (4) of the robot is considered during this trajectory design stage, the neglected angular momentum $\mathbf{L}$ can cause the robot to slip around the gravity vector and rotate the robot about its vertical axis. Using the arms, like humans do during walking, to compensate the angular momentum caused by the joints of the lower body can reduce the overall angular momentum significantly. In this paper an algorithm is presented to generate velocities for the joints using (4) such that the overall angular momentum is reduced.
In the transition from Eq. (2) to Eq. (3) we assumed a high gain position control law and sufficiently accurate joint position tracking. On the physical robot the accuracy of joint tracking can be increased significantly by means of torque feed-forward control. Equation (3) can be used to compute these feed-forward torques for a given walking pattern using inverse dynamics. For the single support, i.e. only one foot has contact to the ground, the contact wrench acting on the swing leg is equal to zero, which leads to a unique solution of the inverse dynamics. In contrast to the single support the system is over-actuated during double support, leading to an infinite number of solutions for the inverse dynamics. To find a solution during this stage one needs to find a criterion, for example the minimization of the contact forces or the internal tensions. An algorithm is presented that solves a quadratic optimization problem to achieve an optimal force distribution of the contact forces between the left and the right leg such that contact forces are minimized. As already mentioned there is only unilateral contact between a foot and the ground, which limits the set of feasible contact wrenches and needs to be considered in the quadratic program.
In both applications we can exploit the recursive calculation of the equations of motion used during the subsystem synthesis giving rise to real-time algorithms that can be used on a physical humanoid robot system.

References

[1] H. Bremer. Elastic Multibody Dynamics: A Direct Ritz Approach. Springer Verlag, Heidelberg, 2008.

[2] J. Mayr, H. Gattringer, H. Bremer. A Bipedal Walking Pattern Generator that Considers Multi-Body Dynamics by Angular Momentum Estimation. In Proceedings of the IEEE/RAS International Conference on Humanoid Robots, pp. 178–182, Osaka, 2012.