# **Energy Optimal Manipulation of an Industrial Robot**

### Thomas Lauß, Peter Leitner, Stefan Oberpeilsteiner, Wolfgang Steiner,

Faculty of Engineering and Environmental Sciences University of Applied Science Upper Austria Stelzhamerstraße 23, 4600 Wels, Austria E-Mail: thomas.lauss@fh-wels.at Phone: +43 (0)50804-44498

### Abstract

The main goal of this contribution is to determine the excitation of an industrial robot, such that the energy consumption becomes a minimum during the manipulation of the tool center point (TCP) from a start position to a given end point. Such problems can be restated as optimization problems where the functional to be minimized consists of the endpoint error and a measure of the energy. The gradient of this functional can be calculated by solving a linear differential equation, called the *adjoint system* [1, 2]. On the one hand the minimum of the cost functional can be achieved by the gradient method where a proper step size has to be found or on the other hand by the BFGS-method where the inverse of the Hessian can be appreciated.

# **1** Problem Definition

At first, let us consider a nonlinear dynamical system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t), \quad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{1}$$

where  $x \in \mathbb{R}^n$  and  $u \in \mathbb{R}^m$  denote the vectors of state and input variables. In our special case the input variables are torques which appear linear on the right side of Equation (1). In combination with a cost functional

$$J = \alpha \int_{t_0}^{t_f} h(\boldsymbol{x}, \boldsymbol{u}, t) \, \mathrm{d}t + \beta S(\boldsymbol{x}_f, t_f) \to \min$$
(2)

which has to be minimized, we obtain an optimal control problem. In Equation (2) the term S is the so called *Scrap-Funtion* which may describe the endpoint error and h is a time-dependent function which contains for example the mechanical power or the quadratic input signal at one point in time. It is also possible to include position and input constraints in the cost functional as a penalty function. The weighting factors  $\alpha$  and  $\beta$  have to be chosen properly.

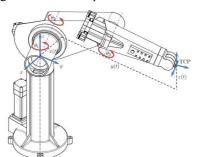


Figure 1: schematics of the six-axis robot

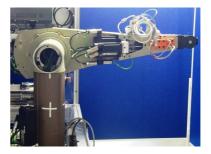


Figure 2: photograph of the six-axis robot

A very common approach in pertinent literature is to minimize the quadratic signal energy (cf. [3])  $h(\mathbf{u}) = \mathbf{u}^T \mathbf{u}$ . The advantage of this method is that the convergence rate of the optimization process is quite good due to the quadratical formulation. On the other side this measure has nothing to do with the real energy, it is rather a mathematical construct which has been established due to good convergence. Hence, it suggests itself that one could take the real mechanical energy into account. Therefore the function  $h(\mathbf{v}, \mathbf{u}) = \mathbf{u}^T f_{\mathbf{u}}^T \mathbf{v}$  has to be minimized, where  $\mathbf{v}$  is the velocity of the appropriate degree of freedom (DOF) and  $f_{\mathbf{u}}$  combines the input signals with the proper DOF.

## 2 Results

The robot which is depicted in Figure 1 is used to test the presented method with the two different definitions of the cost functional. Afterwards the simulation results are verified at a real six-axis-robot which is shown in Figure 2. The system consists of three degrees of freedom,  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  which denote the relative rotation angles of the joints. Due to the complicated structure of the equations of motion and the minor influence on the energy consumption the three wrist joints are fixed. The left diagram of Figure 3 shows the signal energy expenditure of the standard manipulation in comparison to the optimization with respect to the signal energy. For the sake of completeness also the signal energy expenditure of the optimization with respect to the mechanical energy is pictured. As a result the reduction of the signal energy after the optimization process is about 47% with respect to the standard manipulation of the robot control. On the right hand side of Figure 3 the real mechanical energy expenditure is pictured. A reduction of 34% could be achieved if the mechanical energy is taken into account in the cost functional. Hence, the minimal signal energy does not ensure an energy optimal manipulation of the robot.

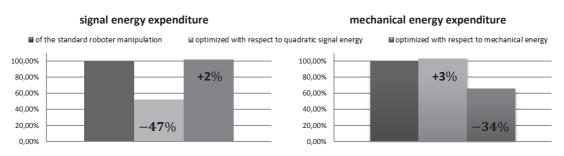
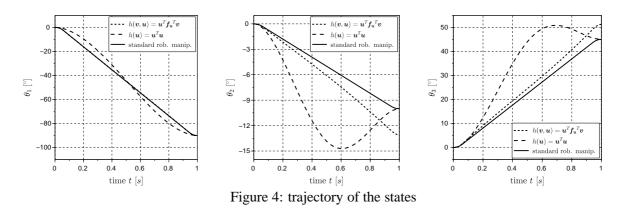


Figure 3: comparison of the energy expenditure

In Figure 4 the joint angles of the optimized motions in comparison to the standard manipulation of the robot are plotted over time. It can be seen that the prescribed end position of the motion, optimized with respect to the mechanical energy, is not met. This angular deviation results in a small endpoint error with a magnitude of 2.49% with respect to the TCP-vector  $[x(t_f), y(t_f), z(t_f)]^T$  which is marked in Figure 1. In practical terms this means that one could reduce the energy expenditure by 34% if the end position is modified slightly. In consideration of the fact that we do not take friction into account, the comparison of the measure versus the simulation is surprisingly good.



#### References

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