One Approach to the Parameterization of Dynamic Models of Serial Mechanisms

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Abstract

Consider an open serial kinematic chain of N rigid bodies connected with lower pair joints (except helical joints) that provide n degrees of freedom (DoF). Canonical equations of motion of such mechanisms are usually derived from the Euler-Lagrange equations. It is well known that the Lagrangian L for a scleronomous n-DoF mechanical system is a quadratic function of generalized velocities

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(\mathbf{q})\,\dot{q}_i\dot{q}_j - P(\mathbf{q}). \tag{1}$$

Here q_i and $\dot{q}_i$ are generalized coordinates and velocities, respectively, a_{ij} is a symmetric positive-definite tensor field of generalized inertia and P is a scalar field which represents the potential energy of gravity. This results in the next equations of motion [1]:

$$\sum_{i=1}^{n} a_{ki}(\mathbf{q})\,\ddot{q}_i + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{kij}(\mathbf{q})\,\dot{q}_i\dot{q}_j + c_k(\mathbf{q}) = Q_k,\ k = \overline{1,n}, \tag{2}$$

where $\ddot{q}_i$ are generalized accelerations, $b_{kij} = \left(\partial a_{ki}/\partial q_j + \partial a_{kj}/\partial q_i - \partial a_{ij}/\partial q_k\right)/2$ are Christoffel symbols of the first kind for generalized inertia tensor, $c_k = \partial P/\partial q_k$ are generalized forces due to gravity and Q_k are generalized non-potential forces, e.g. joint drive torques for robot manipulators.
Using classical parameterization the Lagrangian is completely determined by numerous generally non-linear combinations of the geometrical and inertial parameters of a mechanism, and so are the canonical equations. However, there has been no universal approach to direct calculation of these combinations so far. General-purposed algorithms of dynamic model formulation in conjunction with special techniques of symbolic computations are used to obtain expressions of the equations and perform some simplifications [2, 3]. In this paper a different approach is presented. It is based on the interpretation of the Lagrangian function not as a tensor field but as a vector in the linear space of all continuous quadratic functions of the same form. This space is infinite dimensional but due to the special structure of generalized inertia and potential energy it is possible to find finite dimensional linear subspace $\mathbb{B}$ which certainly contains the Lagrangian L. This space might be constructed as a tensor product of elementary subspaces

$$\mathbb{B} = \mathbb{F}_0 \otimes \mathbb{F}_1 \otimes \ldots \otimes \mathbb{F}_n, \tag{3}$$

which are defined as follows[1]

$$\mathbb{F}_0 = \operatorname{span}\left\{\{1(\dot{\mathbf{q}})\} \cup \left\{\dot{q}_i^2/2, i = \overline{1,n}\right\} \cup \left\{\dot{q}_i\dot{q}_j, i = \overline{2,n}, j = \overline{1,i-1}\right\}\right\}$$

and

$$\mathbb{F}_k = \begin{cases} \operatorname{span}\left\{1(q_k), \cos q_k, \sin q_k, \cos(2q_k), \sin(2q_k)\right\} & \text{if } k\text{-th DoF is rotational,} \\ \operatorname{span}\left\{1(q_k), q_k, q_k^2\right\} & \text{if } k\text{-th DoF is translational.} \end{cases}$$

Thus, the Lagrangian can be represented as a linear combination of the basis vectors of the space $\mathbb{B}$. To number basis vectors we introduce the set $J_0 = \{0, 1\ldots, n(n+1)/2\}$ and the set $I_n = J_1 \times \ldots \times J_n$ of tensor indices where $J_k = \left\{1, \ldots, \dim \mathbb{F}_k\right\}$. Suppose b_m^l ($m \in J_0$, $l \in I_n$) form the natural basis[2] of $\mathbb{B}$, then

$$L = \sum_{l \in I_n}\sum_{m \in J_0} \mu_m^l b_m^l. \tag{4}$$

[1] Hereinafter $1(x)$ means function which equals to 1 for all x from its domain.
[2] It is constructed as a full set of tensor products of vectors from bases of all elementary subspaces $\mathbb{F}_k$.

Let's define *generalized inertial parameters* as the coordinates μ_m^l of the Lagrangian L in the natural basis of space $\mathbb{B}$. Due to the properties of bases, the generalized parameters set determines dynamic model of a mechanism with a fixed set of DoF types completely and uniquely. Total amount of generalized inertial parameters equals to $\dim\mathbb{B} = \left(1+(n^2+n)/2\right)5^{\nu}3^{n-\nu}$, where ν is the number of rotational DoF. Although the dimension of $\mathbb{B}$ is extremely high, a lot of generalized parameters are actually zero (moreover, a large number of them apriori). To work out this problem, the connection between generalized and classical inertial parameters should be analyzed. It is determined by a linear mapping $\mu = \mathrm{C}\mathbf{p}$, where $\mu \in \mathbb{R}^{\dim\mathbb{B}}$ is the column vector of all generalized parameters μ_m^l, $\mathbf{p} \in \mathbb{R}^{10N}$ is the column vector of all classical parameters and C is a matrix of the respective size, which depends on the structure and parameters of constraints. As this matrix is calculated symbolically, generalized parameters which correspond to its zero rows are definitely zero. Additional simplification for a particular mechanism can be performed by discovering the row subspace which is orthogonal to the given classical parameters vector.
If both geometrical and inertial parameters of a mechanism are unknown, the set of those generalized inertial parameters which apriori are non-zero is the minimal set of dynamic model parameters. It is also a structurally identifiable one. The last follows from linear independence of the functions b_m^l, which generate columns of regressor matrix of the energy identification model [1]. If geometrical parameters are known, the minimal parameter set can be significantly reduced. Consider the linear space $\mathbb{P}$ of classical inertial parameters $\mathbf{p}$, which is isomorphic to $\mathbb{R}^{10N}$. Then, the transpose of the matrix C might be thought of as the coordinate matrix of generalized inertial parameters in the standard basis of $\mathbb{P}$. The linear independent rows of C determine the basis $\tilde{\mu} \in \mathbb{R}^r$ of generalized parameters, where $r = \operatorname{rank} \mathrm{C} \leq \min(10N, \dim\mathbb{B})$. It forms the minimal parameter set since the other parameters can be represented as some linear combinations of its elements. This set is also an identifiable one because it coincides with the set of base inertial parameters [1] up to a non-singular linear transformation.
Finally, the canonical equations of motion using generalized inertial parameters might be represented in the following form:

$$\sum_{l \in I_n} b_0^l \left(\sum_{i=1}^{n} a_{ki}^l \ddot{q}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{kij}^l \dot{q}_i \dot{q}_j + c_k^l \right) = Q_k, \tag{5}$$

where $b_0^l \in \mathbb{F}_1 \otimes \ldots \otimes \mathbb{F}_n$, parameters a_{ki}^l are exactly the same as μ_m^l assuming $m = \operatorname{ind}(k,i)$ and $\operatorname{ind}(i,j) = \max(i,j)(\max(i,j)-1)/2 + \min(i,j)$. Parameters c_k^l are equal to μ_0^l up to certain substitutions of the tensor index l, and parameters b_{kij}^l are predefined linear combinations of the certain μ_m^l.
The presented approach offers a new kind of parameterization of dynamic models of serial mechanisms using the set of generalized inertial parameters. Its basis subset is the minimal parameter set of the model. To find generalized parameters set and, hence, its basis, the linear mapping matrix C must be calculated. A recursive computationally efficient algorithm has been developed and is being tested at the moment. The suggested approach also allows direct calculation of the coefficients of canonical motion equations using both ordinary and basis generalized parameters. This can reduce the computational cost of the symbolic formulation of the equations avoiding application of complicated computer algebra methods, in particular, manipulations with trigonometric expressions. An efficient algorithm for generating the equations is under development. Finally, the basis generalized parameters set may be treated as one of the possible choices of the base inertial parameters set, so it can be estimated with one of the existing techniques [1]. In this case, either Equation (4) or (5) can be used for symbolic generation of the corresponding identification model. Further plans include generalization of the presented approach for some types of higher kinematic pairs as well as its adaptation to open kinematic trees.

References

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