Validation of Flexible Multibody Dynamics Beam Formulations using Benchmark Problems

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Abstract

As the need to model flexibility arose in multibody dynamics, the floating frame of reference formulation was developed. Usually, this approach is based on a geometrically linearized formulation and thus can yield inaccurate results when elastic displacements become large. While the use of three-dimensional finite element formulations overcomes this problem, the associated computational cost is overwhelming. Consequently, beam models, which are one-dimensional approximations of three-dimensional elasticity, have become the workhorse of many flexible multibody dynamics codes.

Numerous beam formulations have been proposed, such as the geometrically exact beam formulation, the co-rotational formulation, or the absolute nodal coordinate formulation, to name just a few. New solution strategies have been investigated as well, including the intrinsic beam formulation or the DAE approach. Finally, Lie group concepts are playing an increasing role in the field. Clearly, a systematic comparison of these various approaches is desirable and is the focus of this paper.

Various beam formulations have been assessed by comparing their predictions for four benchmark problems [1]. The first problem is the Princeton beam experiment, a study of the static large displacement and rotation behavior of a simple cantilevered beam under a gravity tip load. The second problem, the four-bar mechanism, focuses on a flexible mechanism involving beams and revolute joints. The third problem investigates the behavior of a beam bent in its plane of greatest flexural rigidity, resulting in lateral buckling when a critical value of the transverse load is reached. The last problem investigates the dynamic stability of a rotating shaft. The predictions of eight independent codes are compared for these four benchmark problems.

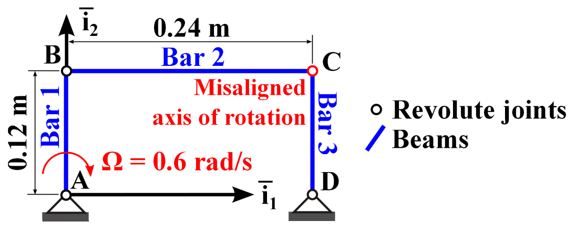

Figure 1: Configuration of the four-bar mechanism.

The second benchmark problem will be described briefly; fig. 1 depicts a flexible four-bar mechanism. Bar 1 is of length 0.12 m and is connected to the ground at point **A** by means of a revolute joint. Bar 2 is of length 0.24 m and is connected to bar 1 at point **B** with a revolute joint. Finally, bar 3 is of length 0.12 m and is connected to bar 2 and the ground at points **C** and **D**, respectively, by means of two revolute joints.

In the reference configuration, the bars of this planar mechanism intersect each other at 90 degree angles and the axes of rotation of the revolute joints at points **A**, **B**, and **D** are normal to the plane of the mechanism. To simulate an initial defect in the mechanism, the axis of rotation of the revolute joint at point **C** is rotated by +5 degrees about unit vector $\bar{\imath}_2$ indicated in fig. 1. The angular velocity at point **A** of bar 1 is prescribed to be $\Omega = 0.6$ rad/s for the duration of the simulation.

Bars 1 and 2 are of square cross-section of size 16 by 16 mm; bar 3 has a square cross-section of size 8 by 8 mm. The three bars are made of steel, whose mechanical characteristics are Young's modulus E = 207 GPa and Poisson's ratio ν = 0.3. These physical properties translate to the sectional stiffness properties listed in table 1. The sectional mass properties are as follows: mass per unit span m_{00} = 1.997 and 0.4992 kg/m, moments of inertia per unit span $m_{22} = m_{33}$ = 42.60 and 2.662 mg·m^2/m for Bars 1 and 2, and Bar 3, respectively.

Table 1: Sectional stiffness properties of the bars

	Axial S [MN]	**Shearing** K_{22} [MN]	**Shearing** K_{33} [MN]	**Torsional** H_{11} [N·m^2]	**Bending** H_{22} [N·m^2]	**Bending** H_{33} [N·m^2]
Bar 1 & 2	52.99	16.88	16.88	733.5	1131	1131
Bar 3	13.25	4.220	4.220	45.84	70.66	70.66

A comprehensive set of results for the four proposed benchmark problems will be presented. The predictions of each of the eight codes used in this effort are found to be in excellent agreement with each other. The focus of this paper is the assessment of the performance of the eight codes. Two fundamental aspects of the formulation of beam elements for flexible multibody dynamics will be investigated: the spatial and the temporal converge of the codes.

The spatial convergence of the codes is assessed easily by monitoring the error as a function of mesh size for various types of elements. For instance, fig. 2 shows the error in the tip rotation of the Princeton beam (the first benchmark problem) as a function of the number of degrees of freedom for linear, quadratic and cubic elements. As expected, the convergence increases with the order of the element.

The temporal convergence of the time integrators is assessed easily by monitoring the error as a function of time step size for various types of time integration schemes. For instance, fig. 3 shows the error in the mid-span bending moments of the rotating shaft (the fourth benchmark problem) as a function of the time step size for the generalized-α time integrator.

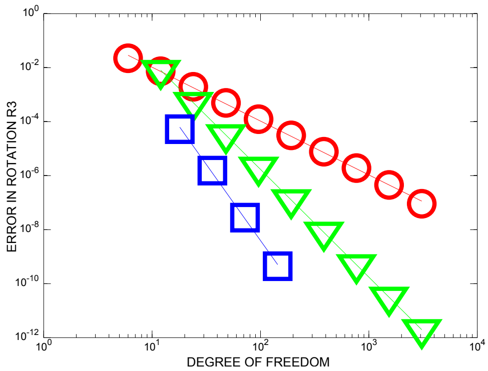

Figure 2: Logarithmic plot of error in rotation R3 versus DOF for three types of elements. Linear(◯), quadratic (△), and cubic (□) element.

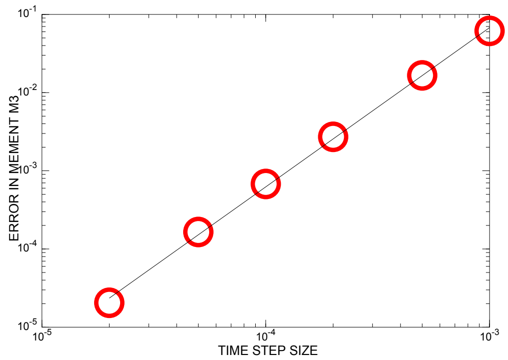

Figure 3: Logarithmic plot of error in mid-point bending moment M3 versus time step size. The generalized-α method is used as a time integrator.

The spatial and temporal convergence characteristics of the eight codes used in this effort will be presented in the final paper.

References

[1] O.A. Bauchau, G.Y. Wu, Betsch P., A. Cardona, J. Gerstmayr, B. Jonker, Masarati P., and V. Sonneville. Validation of flexible multibody dynamics beam formulations using benchmark problems. In *Proceedings of the Third Joint International Conference on Multibody System Dynamics*, Busan, Korea, June 30-July 3, 2014.