Benchmarking of augmented Lagrangian and Hamiltonian formulations for multibody system dynamics

Francisco González[*], **Daniel Dopico**[*], **Roland Pastorino**[*†], **Javier Cuadrado**[*]

[*]Laboratorio de Ingeniería Mecánica
University of La Coruña
Mendizábal s/n, 15403 Ferrol, Spain
[f.gonzalez, ddopico,
rpastorino, javicuad]@udc.es

[†] PMA division,
Department of Mechanical Engineering
KU Leuven
Celestijnenlaan 300b, 3001 Leuven, Belgium
roland.pastorino@mech.kuleuven.be

Abstract

Penalty-based augmented Lagrangian formulations represent a robust and efficient way of carrying out the forward-dynamics simulation of multibody systems. The consideration of the kinematic constraints that affect the motion of a multibody system usually leads to the need for expressing its dynamics equations as a system of DAE's. If a mechanical system is described with a set of n generalized coordinates $\mathbf{q}$, subjected to m kinematic constraints $\mathbf{\Phi}$, the equations of motion can be expressed as

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{c} &= \mathbf{f} + \mathbf{f}_c \\ \mathbf{\Phi} &= \mathbf{0} \end{aligned} \tag{1}$$

where $\mathbf{M}$ is the $n \times n$ mass matrix, $\mathbf{c}$ contains the Coriolis and centrifugal forces, and $\mathbf{f}$ and $\mathbf{f}_c$ are the applied and constraint forces, respectively. Following a Lagrangian approach, the constraint reactions can be expressed as $\mathbf{f}_c = -\mathbf{\Phi}_\mathbf{q}^\mathrm{T}\boldsymbol{\lambda}$, where $\mathbf{\Phi}_\mathbf{q}$ is the $m \times n$ Jacobian matrix of the constraints and $\boldsymbol{\lambda}$ is a set of m Lagrange multipliers.

The augmented Lagrangian formulation described in [1] transforms system (1) into a set of index-1 DAE's, which is then solved via an iterative update of the system accelerations and Lagrange multipliers. Similar formulations based on Hamilton's canonical equations were subsequently introduced in [2]. The algorithms described in [1] and [2] can handle the forward-dynamics simulation of systems with a rank deficient Jacobian matrix $\mathbf{\Phi}_\mathbf{q}$, which enables them to deal with redundantly constrained mechanisms and systems undergoing singular configurations. It is stated in [2] that the methods based on canonical equations are more robust than their classical augmented Lagrangian counterparts and do not show pathological behaviour in singular configurations. However, canonical formulations have received comparatively less attention in the literature (e.g. [3], [4]) since they were first published.

In this research, a systematic study of the performance of these formulations has been carried out. Special attention was paid to their behaviour in the proximity of singular configurations. The comparison of the different algorithms was done using test examples from the IFToMM benchmark problem library [5].

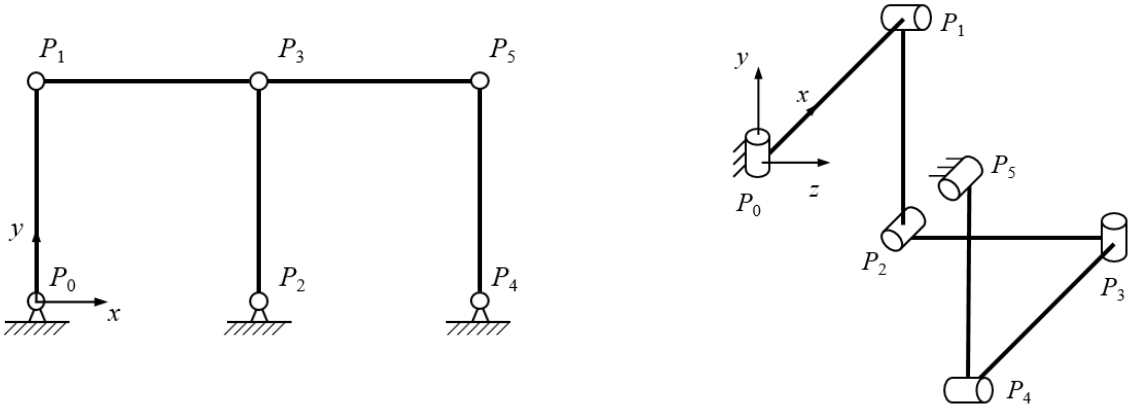

Figure 1: Some benchmark problems used in the numerical simulations: (a) Double four-bar linkage (b) Six-link rectangular Bricard mechanism

Figure 1 shows two benchmark problems used in this study. The first one is a double four-bar linkage. This mechanism is in a singular configuration when the rods connecting points P_1, P_3, and P_5 to the ground are aligned with the global x axis. The Jacobian matrix $\mathbf{\Phi_q}$ experiences a sudden loss of rank in these configurations. The second one is a six-link rectangular Bricard mechanism. This system is redundantly constrained for the entire range of its motion; consequently, its Jacobian matrix is permanently rank deficient.

Both augmented Lagrangian and Hamiltonian formulations were able to successfully deal with the dynamics simulation of the Bricard linkage, while numerical difficulties were observed in some cases when the four-bar linkage passed through a singular configuration, which sometimes led to the failure of the numerical simulation. The comparison of these mechanical systems highlights the fact that a rank-deficient Jacobian matrix is not the cause of these numerical problems. The leading matrix of the augmented Lagrangian and Hamiltonian formulations in [1] and [2] is always symmetric and positive-definite, even when the Jacobian matrix of the constraints is rank-deficient.

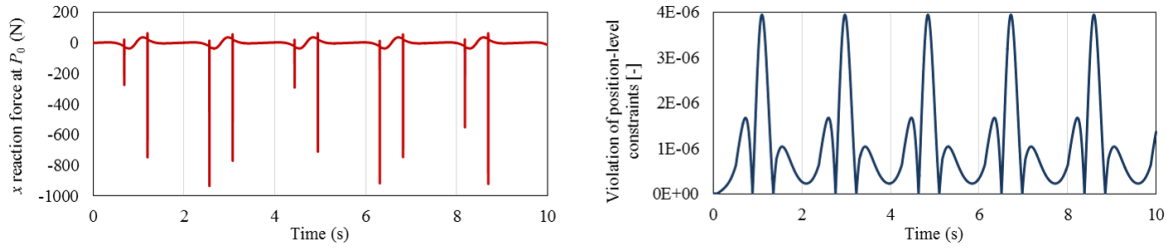

Figure 2: Time-history of the x reaction force at point P_0 (left) and the configuration-level violation of constraints during a 10 s simulation of the motion of the double four-bar linkage

Simulation results suggest that the cause of the numerical problems is the introduction of impact forces in the constraint reactions due to the imperfect fulfilment of the kinematic constraints. Fig. 2 shows the time-history of the x reaction force at P_0 and the violation of constraints for a 10 s motion of the four-bar linkage. While a certain violation of constraints exists during the whole motion, the reaction plot in Fig. 2 only shows impacts at certain instants, when the system is near a singular configuration. Factors such as the type and parameters of the numerical integrator, the values of the penalty and stabilization parameters of the formulation, and the integration step-size can significantly change the behaviour of the simulation. The effect of these factors on the efficiency and reliability of the algorithms has been compared in a series of simulations with the above mentioned benchmark examples.

References

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