On the FE Modeling of Active Attenuation of Vibrations by Controlling Relative Motion of Selected Masses

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Abstract

The attenuation effects in a vibrating system with almost absent internal/external friction and no possibility of using passive dampers can be generated by imposing a precisely synchronized relative motion of some of its components. The dynamic interaction between the main body and the moving components triggers the Coriolis type forces that are in fact capable of producing both attenuation and amplification effects in every cycle of the system's vibrations [1]. However, by carefully controlling the relative motion's phase and frequency, it is possible to make the attenuation effects prevailing. This clearly provides a unique means for actively reducing vibrations in the system.

The 'standard' finite element (FE) analysis of such a problem appears to be challenging, mostly because it requires imposing complex geometrical constraints on the relative motion of the considered components. Nevertheless, such an approach was used, for example, in vehicle-bridge or vehicle-rail tract simulations [2] that typically involve only constant relative velocities (i.e. not controlled). However, for the problems in which the moving component is treated as a controller (with a desired motion pattern imposed as input to analyze the system's response as output), the approach in [2] is essentially inapplicable. A 'new' FE procedure presented here allows handling an arbitrary pattern of the moving components, and therefore is suitable for control purposes.

Consider an arbitrary elastic system (Fig.1a) that is modeled with sufficient accuracy by the elements appropriate to simulate its vibrations. A small component of mass m is forced to move along a path AB with coordinate $s(t)$ relative to the system. The path itself is modeled by a guiding beam attached to the vibrating body at nodes (Fig 1b). The properties of the beam represent either a real guiding rail or they may be fictitious. The moving mass interacts only with the beam, which then transfers the forces to the system. The beam element being currently traversed by m is referred to as the composite element. This element, as well as the forces related to the relative motion (referred to as the Coriolis forces), are indicated in Fig. 1c.

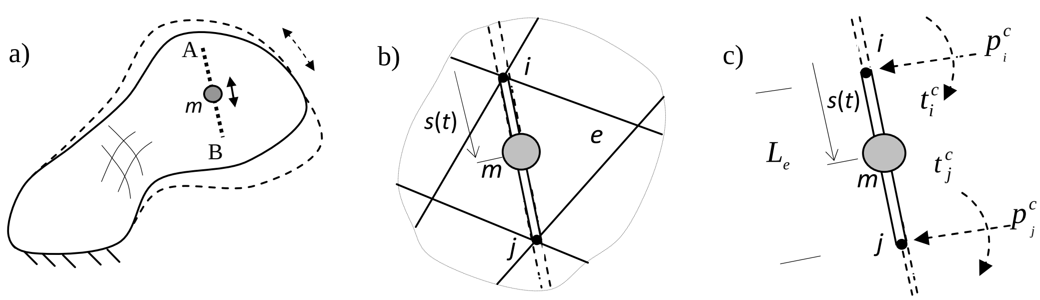

Figure 1: The vibrating system (a) the guiding beam (b), and the composite element (c)

Due to the relative motion only the mass matrix of the composite beam is time-dependent. This makes, however, the mass matrix for the whole system also time-dependent and substantially complicates its analysis. As shown in [3], in order to identify directly the Coriolis forces in the composite element, the inertia forces for this element are considered in the form:

$$\frac{d}{dt}[M_e(t)\dot{u}_e] = M_e(t)\ddot{u}_e + C_e\dot{u}_e = M_e(t)\ddot{u}_e + f_e^c \tag{1}$$

where $C_s = m\dot{s}\frac{\partial}{\partial s}(N^T N)$ can be treated as an 'instantaneous' damping matrix (where N=N(s(t)) are the values of the beam's shape functions at the current mass location). The term $f_e^c = C_e\dot{u}_e$ defines the nodal Coriolis forces indicated in Fig 1c. The matrix C_s and vector f_e^e depend explicitly on the current relative velocity $\dot{s}$ of the mass and implicitly on its current position between the nodes of the element (the position is hidden in functions N).

Formula (1) permits writing the element equation in the following forms (in order to concentrate on the active attenuation any passive damping is omitted):

$$M_e(t)\ddot{u}_e + C_e\dot{u}_e + K_e u_e = F_e \qquad \text{or} \qquad M_e(t)\ddot{u}_e + K_e u_e = F_e - f_e^c \tag{2}$$

The first form indicates how the relative motion relates to periods of attenuation ($C_s > 0$ if $\dot{s} > 0$) and to periods of amplification ($C_s < 0$ if $\dot{s} < 0$), while the second form is more suitable for the practical simulation due to the fact that all the LHS terms can be routinely handled by typical FE software (such as ANSYS, for example), while f_e^e on the RHS can be easily calculated and added at each time step of the integration procedure. The implementation and accuracy of the procedure will be discussed in details.

As illustration the test case of using relative motion of two small masses to attenuate vibrations of a frame is presented in Fig. 2. An effective active damping ratio of about 2.7% was generated.

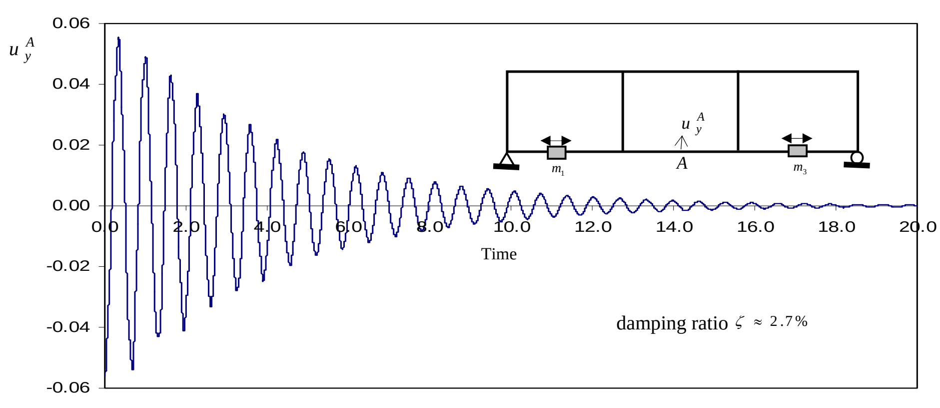

Figure 2: The response of vibrating frame controlled by motion of three masses.

References

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