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Formulation of constraints in contact dynamics

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Abstract

Contact interactions are ubiquitous in a variety of mechanical systems, and are usually characterized as kinematic constraint equations when rigid body approaches are employed to model the forward dynamics of the systems. For simple problems such as a disk or a ball contacting with a plane, it is easy to explicitly write out the expression of the constraint equations, then they can be incorporated into the dynamics of the systems. However, as the contact motion occurs between two bodies with surfaces in complex geometry, how to formulate the contact constraints and embed them into dynamics is still a tough issue. The reason is that the contact constraint strictly depends on the parameters in describing the surface geometries of the contacting bodies, whose values vary with the relative motion at the contact point.

While studies on mechanical systems subject to constraints started over long time ago, discussion on the kinematic constraints in contact motion is relatively recent, and is motivated mainly by the development in robot researches. Cai and Routh[1] studied the relative motion of two contacting bodies in point contact. Montana[2] derived a differential-geometric model of the rolling constraint between general bodies, and discussed applications to robotic manipulation. Li and Canny[3] discussed the special kinematics of two rigid bodies rolling on each other. Jia and Erdmann[4] derived the contact kinematics directly from the absolute velocities of the contacting bodies instead of their relative velocities at the contact. In order to obtain a dynamic model of the system subject to contact motion, Sarkar etc.[5] adopted the Montana's definition of the configuration to derive the velocity and acceleration equations for three-dimensional rolling contacts.

All the aforementioned work is in general on the basis of the work given by Pars[6], who stated that the relative motion between two rigid bodies in a point contact can be described by five independent variables. Using methods from differential geometry, together with the five independent variables, these authors provided a concise form to determine the values of surface parameters at the contact point. Since the objective of these studies was to establish an approval control methodology through kinematic analysis, little attention was paid on how to incorporate the contact-induced constraints into the forward dynamics.

Taking contact directly as constraint functioned by system's generalized coordinates and velocities is concerned by researchers working on the field of multibody systems. Pfeiffer[7] and Glocker[8] used the theory of differential geometry to derive the contact constraint equations when two bodes with convex contours keep contacting together. By introducing the quantities related to the local properties of surface geometries and the relative motion, they derived the relative velocities in normal and tangential direction of the contact, then provided a formulation for the relevant accelerations. However, the contact constraints between two bodies with shapes in other geometries, such as points and spatial curves, are not discussed in their studies.

In this paper, we discuss the formation of the constraints in a point contact that occurs between two bodies with general contour shapes. By classifying the possible geometries of the contacting surfaces, we establish the geometric conditions that should be satisfied during the contact motion. Note that the geometric conditions consist of the variables in describing system configurations and the surface shape of the two contacting bodies. The resulting geometric constraint usually cannot be formulated explicitly. Often, an integral process is needed to determine the values of the parameters in describing the positions of the contact point on the contacting surfaces. A numerical approach to find these values of parameters and their derivations are present in this paper. We can then numerically determine the coefficients before the Lagrange multipliers needed in embedding these geometric constraint equations into dynamics. In addition, we also discuss the formulation of the nonholonomic constraints that may appear at the instant when the relative velocity at the contact point vanishes.

In summary, we will classify the possible geometries between two contacting bodies and establish the geometric conditions that should be satisfied by the geometric constraints. Then, a numerical procedure to determine the values of the surface parameters and their derivations will be presented. Also, we will discuss the formulation of the nonholonomic constraints at the contact point. The constraints in two well-known examples, a thin disk and a ball rolling purely on a horizontal rough plane, will be formulated following the procedure of the method presented in this paper.

References

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