Automated natural coordinate selection for fast symbolic–numeric dynamic simulation

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Abstract

The combination of symbolic and numeric techniques affords promising advantages for the dynamic simulation of mechanical systems. Symbolic tools using exact arithmetic are valuable when predefined algebraic expressions or moderately sized systems must be manipulated. On the other hand, when large problems must be solved or a preprocessing stage is undesirable, numeric implementations based on floating-point numbers tend to be more effective. In this work, we present a method for the automatic modeling of multibody systems using natural coordinates and combined symbolic–numeric techniques, as a generalization and significant advancement of the work previously reported by Uchida et al. [1].

The natural coordinates are partitioned into independent and dependent sets, and *Gröbner bases* are used to triangularize the resulting nonlinear constraint equations in a way that resembles the Gaussian elimination procedure for systems of linear equations. The resulting triangular system is used to solve the *finite displacement problem* [2] analytically, recursively calculating the dependent coordinates ($\mathbf{q}^d$) given the independent ones ($\mathbf{q}^i$) at every time step of the simulation. The benefits of this approach are threefold. First, constraint equations are satisfied to within machine precision at all times and, thus, energy conservation is facilitated. Secondly, the finite displacement problem is solved in a fixed amount of time, which is ideal for real-time applications. Finally, when constraints are formulated using *natural* or *fully Cartesian coordinates* [3] (as opposed to absolute or joint coordinates), the triangularization process is particularly efficient due to the maximally quadratic nature of constraints in natural coordinates [1].

In the presented algorithm, the selection of natural coordinates (which is often done heuristically in the literature) and the generation of a valid set of triangular solutions are fully automated. We address the following computational challenges:

- Detection and resolution of motion singularities;
- Generation of multiple triangular systems for different coordinate partitions;
- Management of redundant (but compatible) constraint equations;
- Optimization of the coordinate selection towards fast dynamic simulation; and
- Interfacing between symbolic and numeric computational tools.

We generate an ensemble of triangular systems, each being associated with a different set of independent and dependent coordinates. Upon solving the position-level kinematics symbolically and exporting the ensemble of triangular systems, the embedding [4] or matrix-R [5] technique is applied, which involves assembling the Jacobian matrix of constraint equations symbolically and computing a basis of the Jacobian matrix nullspace numerically. This basis is then used to solve the velocity-level kinematics problem and, with the other dynamic terms, to form the state-space dynamic equations, with one differential equation per degree of freedom (DOF). An overview of the algorithm is shown in Figure 1.

The validity of the coordinate partitioning is monitored in real time through the column rank of the dependent Jacobian matrix ($\mathbf{\Phi}_\mathbf{q}^d$), which should always equal $m = n - f$. Note that $\mathbf{\Phi}$ is the vector of constraint equations, n is the number of natural coordinates, and f is the number of DOFs. At the beginning

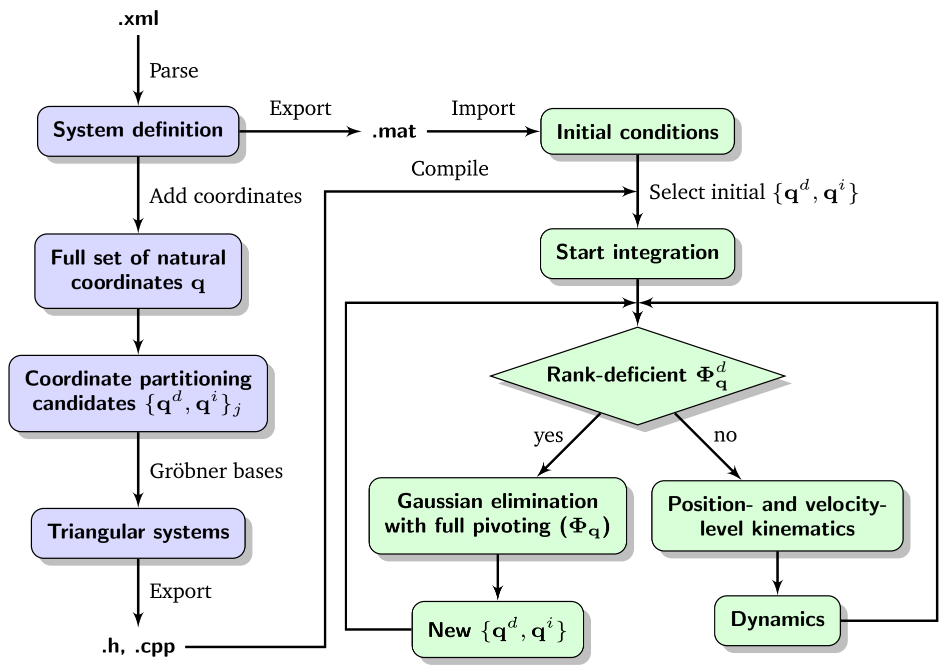

Figure 1: Overview of symbolic (left) and numeric (right) components of our algorithm for the automatic modeling of multibody systems using natural coordinates.

of the simulation, the lowest-cost triangular system is selected. Whenever $\mathbf{\Phi}^d_{\mathbf{q}}$ becomes ill-conditioned due to a deterioration of the coordinate partition, the algorithm performs Gaussian elimination with full pivoting on the Jacobian matrix, yielding a new set of independent (and dependent) coordinates. The corresponding triangular system is then adopted for solving the finite displacement problem.

Three numerical examples are used to present the theory and validate the modeling environment. First, a 1-DOF pendulum is used throughout the theoretical development to describe the proposed approach. A closed-loop, 1-DOF, revolute–spherical–cylindrical–revolute (RSCR) mechanism is then analyzed. Finally, a 2-DOF landing gear system is simulated.

Novel ideas and implementation details are presented and evaluated for the general-purpose symbolic–numeric simulation of multibody systems using natural coordinates and Gröbner bases. The entire modeling process, from the system definition to the dynamic simulation, by way of position- and velocity-level kinematics, is thoroughly addressed.

References

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