Interpolation and Truncation Model Reduction Techniques in Coupled Elastic Multibody Systems

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Abstract

Many technical systems are constructed in a modular fashion. The division of tasks leads to systems with components that can be separated and reused. Consider for example a robot that can be equipped with exchangeable end effectors. The gross motion of the end effector will be provided by the movement of the robot arms but the specific task of the robot is often performed by a specially designed end effector. Elastic multibody systems (EMBS) are very well suited to describe modular systems. However, certain criteria have to be met if we want to reuse single modules in the simulation. The dynamic behavior of each single elastic body is subject to the surrounding environment and boundary conditions. The goal is therefore to find a representation of the elasticity of each body that can be used in different settings.

The elasticity of single bodies can be modeled with the finite element method (FEM). In order to keep the computational burden at an acceptable level, model order reduction is performed. In this contribution, the floating frame of reference approach is used, which allows for projection based, linear model order reduction (MOR).

The FEM approach delivers a second order ordinary differential equation for the elastic quantities

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{f}(t), \quad \boldsymbol{M},\boldsymbol{K}\in\mathbb{R}^{N\times N}, \quad \boldsymbol{x}(t),\boldsymbol{f}(t)\in\mathbb{R}^{N} \tag{1}$$

with constant mass and stiffness matrices and the vector of applied forces $\boldsymbol{f}(t)$. We seek a subspace $\mathscr{V}$ spanned by the columns of $\boldsymbol{V}\in\mathbb{R}^{N\times r}$ such that $\boldsymbol{x}(t)\approx\boldsymbol{V}\boldsymbol{q}_{\mathrm{r}}(t)$. With Galerkin conditions involved, we solve the dynamic system of dimension r in the reduced generalized coordinates $\boldsymbol{q}_{\mathrm{r}}$

$$\boldsymbol{M}_{\mathrm{r}}\ddot{\boldsymbol{q}}_{\mathrm{r}}(t)+\boldsymbol{K}_{\mathrm{r}}\boldsymbol{q}_{\mathrm{r}}(t)=\boldsymbol{f}_{\mathrm{r}}(t), \quad \boldsymbol{M}_{\mathrm{r}},\boldsymbol{K}_{\mathrm{r}}\in\mathbb{R}^{r\times r}, \quad \boldsymbol{q}_{\mathrm{r}}(t),\boldsymbol{f}_{\mathrm{r}}(t)\in\mathbb{R}^{r} \tag{2}$$

with

$$\boldsymbol{M}_{\mathrm{r}}=\boldsymbol{V}^T\boldsymbol{M}\boldsymbol{V}, \quad \boldsymbol{K}_{\mathrm{r}}=\boldsymbol{V}^T\boldsymbol{K}\boldsymbol{V}, \quad \boldsymbol{f}_{\mathrm{r}}=\boldsymbol{V}^T\boldsymbol{f}. \tag{3}$$

An approximate solution of Equation (1) can be reconstructed based on the integration of the low-dimensional differential equation. The quality of the approximation depends on the subspace $\mathscr{V}$. The basis for this subspace is determined by the model reduction algorithm.

Many of the MOR methods used in this context, see [1] for an overview, can be classified in two categories, truncation methods and interpolation methods. Truncation methods first transform the system into a certain representation. Then, single generalized coordinates respectively states of the system, which are considered unimportant, are removed. Methods of this category include amongst others classical modal truncation and the many variants of balanced truncation [1]. In contrast, interpolation methods match the so-called moments of the transfer matrix of the dynamical system at certain shifts or interpolation points. Guyan condensation [2] and block-Krylov methods [3] are typical variants.

It will be shown that for connected systems, the moment matching conditions which are introduced with interpolation methods for single components also hold for the assembly. At the same time, error bounds that arise in balanced truncation methods do not hold anymore after the introduction of connections to the environment. This behavior can e.g. be observed for a simple beam model as shown in Figure 1. The beam is clamped at one end and excited at the other, where measurements are also taken. After incorporating a stiff spring, connecting the middle node of the beam to the ground, the approximation quality worsens due to the altered stiffness matrix. Note that in both cases, the middle node was also considered in the reduction to be a force input and the originally obtained projection matrix is kept and reused for the modified system. While moment-matching properties are retained (left side), the error

induced by balanced truncation grows massively in comparison to the unaltered system. With frequency-weighted balanced truncation, however, excellent results can be achieved if no connections are introduced [3].
The combination of both categories of MOR methods is e.g. realized in the popular reduction scheme by Craig and Bampton [4], which uses static Guyan condensation and modal truncation. The modes of vibration in the Craig-Bampton scheme can be replaced with different mode sets [5, 6]. In this contribution, it will be shown that the combination of interpolatory and truncation-based MOR can be generalized and the use of Gramian-based truncation methods is suited to greatly improve reduction results.

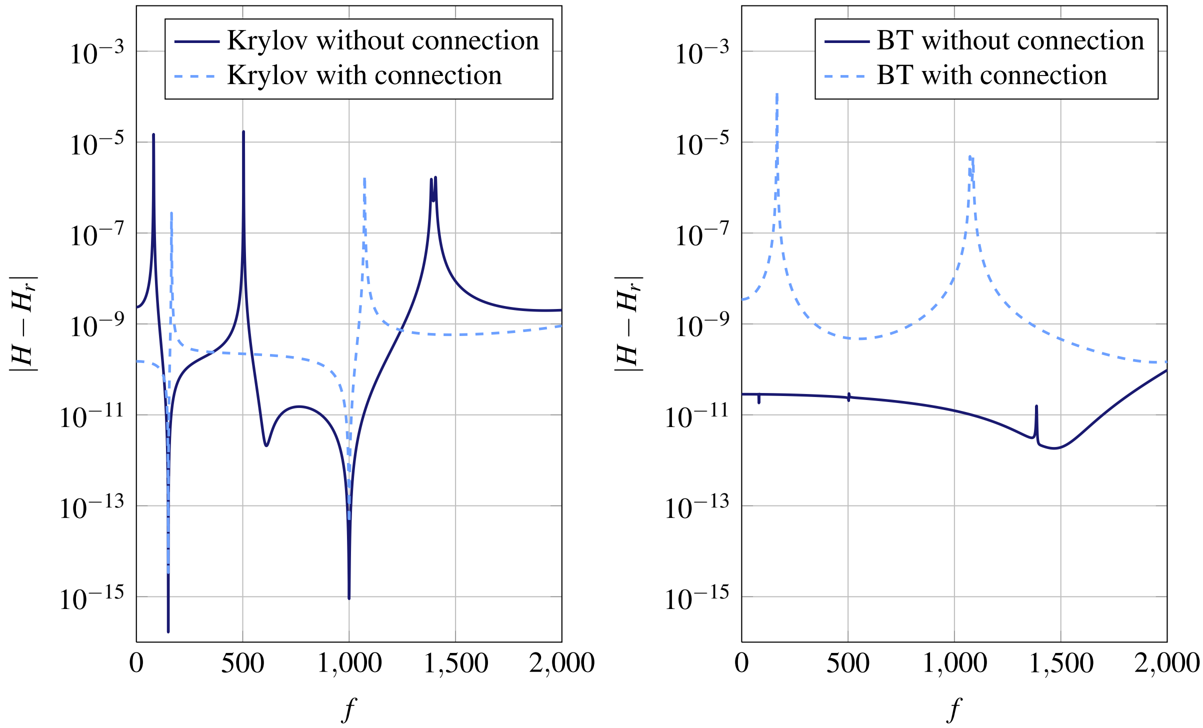

Figure 1: Approximation error for the transfer function of a beam for block-Krylov moment matching (left) and second order frequency weighted balanced truncation (right) with (- - -) and without (——) altered stiffness matrix.

References

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