Planar rigid-body motion identification by tracking pressure points of three ellipses along straight lines

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Abstract

Marker-based motion capture systems are often used to determine the kinematics of the skeletal motion of a human body. However, they are known to be affected by skin movements which can cause significant errors of measurement. In this paper we analyze a method for reconstructing bone motion by tracking the pressure points of given bone landmarks on camera-tracked pressure foils [1], [2]. In the spatial case, the contact-relevant regions of bone landmarks (e.g. epicondyles) can be approximated by ellipsoids, and the pressure points are given by plane coordinates of the pressure foils with respect to a moving rigid plane. In the planar case, which is analyzed in this paper, the ellipsoids become ellipses and the foil pressure sensors become lines (Fig. 1). Assuming that the pressure points of three bone landmarks are tracked, and that lines as well as distances s_i along the lines of the pressure points are given, the question is which poses of the rigid body may be rescued from the given measurements. Note that as the compression of the soft tissue between pressure foils and bone landmarks is unknown, the distance along the common normal of pressure line and bone landmark profile is a dependent variable. Thus the problem is very similar to the 3$\underline{\text{P}}$PR planar parallel manipulator, analyzed in [3], where the legs can be viewed as circle contacts with the lines and the centers of the revolute joints. As shown in [3], for given lines and distances s_i along the lines, there are always exactly two possible poses of the center body (termed "manipulator"). Thus the problem analyzed here is a generalization of [3] by exchanging the three circles by three ellipses. As will be seen, this leads to completely different qualitative solutions.

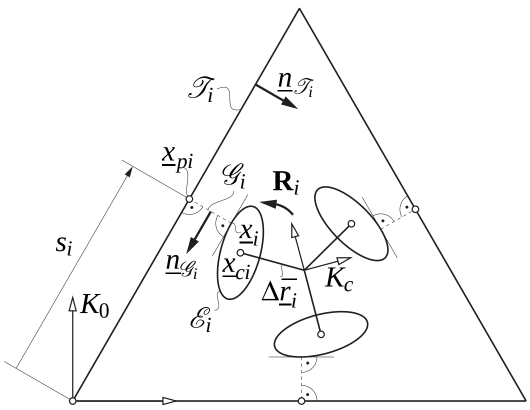

Figure 1: 2D pose detection of rigid body using pressure point tracking along given lines

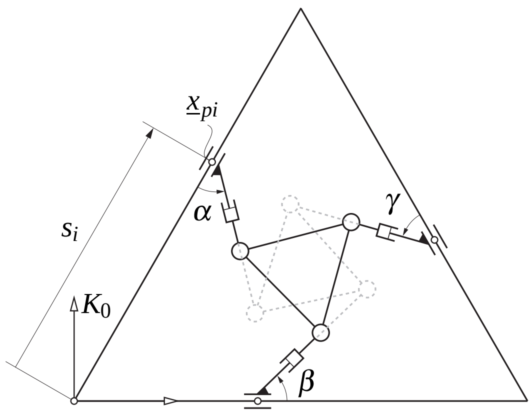

Figure 2: Sample configuration of 3$\underline{\text{P}}$PR parallel manipulator [3] with its two possible solutions

The constraint equations of the system shown in Fig. 1 can be computed as follows

$$(\underline{x}_i - \underline{x}_{ci})^{\mathrm{T}}\,\mathbf{R}\mathbf{R}_i\,\mathbf{A}\,(\mathbf{R}\mathbf{R}_i)^{\mathrm{T}}\,(\underline{x}_i - \underline{x}_{ci}) = 1 \tag{1}$$

$$\mathbf{R}\mathbf{R}_i\,\mathbf{A}\,(\mathbf{R}\mathbf{R}_i)^{\mathrm{T}}\,(\underline{x}_i - \underline{x}_{ci}) = \lambda_i\,\underline{n}_{\mathscr{T}_i} \tag{2}$$

$$(\underline{x}_i - \underline{x}_{pi})^{\mathrm{T}}\,\underline{n}_{\mathscr{G}_i} = 0 \tag{3}$$

for $i = 1..3$. Eqn. (1) represents the equation of an ellipse $\mathscr{E}_i$ with center point $\underline{x}_{ci}$ and the 2×2 diagonal matrix $\mathbf{A} = \mathtt{Diag}(\frac{1}{a^2}, \frac{1}{b^2})$ with the semi-major axis a and the semi-minor axis b, which is rotated about $\mathbf{R}\mathbf{R}_i$ w.r.t an inertial system K_0. $\mathbf{R} = \mathtt{Rot}[z, \varphi_c]$ defines the pure rotation φ_c of the rigid body w.r.t. K_0, whereas $\mathbf{R}_i = \mathtt{Rot}[z, \alpha_i]$ defines the rotation α_i of the ellipse w.r.t. the rigid body frame K_c. $\underline{x}_i$ describes

an arbitrary point on the surface of $\mathscr{E}_i$. The constraint equation (2) implies that the gradient of the ellipse $\mathscr{E}_i$ at point $\underline{x}_i$ should be parallel to the normal vector $\underline{n}_{\mathscr{T}_i}$ of line $\mathscr{T}_i$, where λ_i is an arbitrary scalar. Finally Eqn. (3) constrains $\underline{x}_i$ to be on the line $\mathscr{G}_i$ passing through the specified point $\underline{x}_{pi}$ and is normal to $\underline{n}_{\mathscr{G}_i}$. The ellipse centers result as $\underline{x}_{ci} = \underline{x}_c + \mathbf{R}\Delta\underline{\overline{r}}_i$, where $\underline{x}_c = [x_c, y_c]^{\mathrm{T}}$ are the coordinates of the rigid body center and $\Delta\underline{\overline{r}}_i$ is the vector from the body center to the ellipse center in body-fixed coordinates.
In order to find all possible solutions of Eqns. (1) to (3), Groebner bases (see chapter 3 of [4]) were used to eliminate all intermediate variables such as λ_i and generate reduced polynomials in terms of the three unknown pose variables (x_c, y_c, φ_c) of the rigid body frame K_c. This was achieved after introducing auxiliary variables s and c for the trigonometric functions $\sin\varphi_c$ and $\cos\varphi_c$ with additional constraint $s^2 + c^2 = 1$ and using the Gröbner base library of MAPLE with "PLEX" term order $c \prec s \prec y_c \prec x_c$.
As a result, 32 solutions where found for the symmetric case of Fig. 1, of which 16 were complex and 16 were real. Fig. 3 shows an example of 8 pairs of real solutions. A pair consists of two solutions with the same position (x_c, y_c) of frame K_c. For each solution, the contact point at the line and its corresponding point at the ellipse are marked with a different marker for each line. Note that the solutions are genuinely different, i.e. that they do not result from each other by cyclic transformations, as the pressure point of each line is only associated to one unique ellipse. However, for the present application, only one solution is physically meaningful, as the ellipse contact point must face the corresponding line (black configuration in Fig. 3a). For a non-symmetrical case with unequal ellipses, a total of different 64 solutions were found. However, the number of real solutions for the general case is still an open issue. The results are also being generalized to the 3D case of pressure points between ellipsoids and planes.

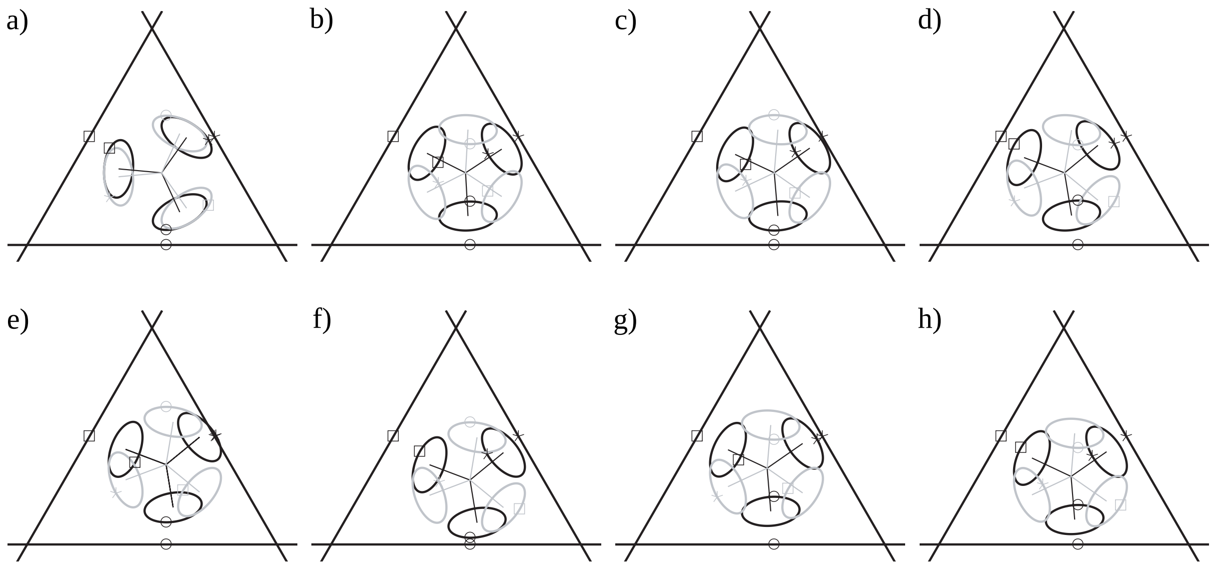

Figure 3: Example of 8 pairs of real solutions (black and gray, respectively) with equal center points

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