Kinematics and Dynamics of Tree-Topology MBS without Body-Fixed Reference Frames

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Kinematics modeling of multibody systems (MBS) heavily involves the definition of body-fixed reference frames (RFR). This not only makes the actual modeling process tedious but also increases the complexity of MBS dynamics simulation codes.

The motion of a rigid (or of the node of a discretized flexible) body is represented by the motion of body-fixed RFR, and the kinematics and dynamics is intrinsically formulated in terms of the representing RFRs [1, 7]. Body-fixed RFRs are further required in order to describe inertia properties. Besides body-fixed RFRs, the MBS kinematics modeling usually involves body-fixed joint frames (JFR). In the relative coordinates MBS modeling approach the latter only serve to describe the relative joint motions. Hence there is no formulation without the (implicit) use of RFRs. But this does not necessarily mean that the explicit definition of body-fixed RFRs is an indispensable step in the modeling of MBS.

A formulation without body-fixed RFRs is one that does not involve explicit definition of body-fixed frames to express the kinematics and the inertia data of an MBS. In this paper a relative coordinate formulation for tree-topology MBS is presented that does not require explicit introduction of *any* body-fixed RFR. It builds upon the previous work presented in [4]. The formulation only involves a single spatial inertial frame (IFR) to model all kinematic and dynamic properties of the MBS. It only requires the joint kinematics (axis and position vector) as well as the inertia tensors w.r.t. the spatial inertial frame in a reference configuration the MBS. That is, the inertia tensors of all rigid bodies are expressed w.r.t. a virtual body-fixed reference frame that coincides with the spatial inertia frame in the reference configuration. All body-fixed frames appearing in the formulation are deduced from the space-fixed IFR in a reference configuration. If a reference configuration of the MBS is known than these joint frames can indeed be deduced from the joint geometry expressed in the IFR making again local frames dispensable.

The theory of screws, respectively Lie transformation groups, provides the mathematical foundation for the presented formulation [6]. Invariance of rigid body motions, and thus of screw entities, w.r.t. a change of frames allows expressing the kinematics of an MBS in a consistent way. In particular the configuration of a rigid body i within a kinematic chain is given as [2, 3, 5, 7, 8]

$$\mathbf{C}_i(\mathbf{q}) = \exp(\widehat{\mathbf{Y}}^{\mathrm{s}}_1 q^1) \cdot \exp(\widehat{\mathbf{Y}}^{\mathrm{s}}_2 q^2) \cdot \ldots \cdot \exp(\widehat{\mathbf{Y}}^{\mathrm{s}}_i q^i)\mathbf{m}_i \tag{1}$$

in terms of the joint screw coordinates $\mathbf{Y}^{\mathrm{s}}_j \in se(3)$ exclusively expressed in the spatial IFR, and the reference configuration $\mathbf{m}_i \in SE(3)$ at the zero reference $\mathbf{q} = \mathbf{0}$.

Avoiding the explicit introduction of body-fixed reference frames significantly simplifies the MBS modeling. This is not only beneficial for manual modeling but also gives rise to much simpler MBS codes. The essential features and advantages of this approach are discussed for tree-topology MBS, and the approach is briefly extended for MBS with kinematic loops. It is demonstrated for various examples including robotic manipulators.

References

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