# An Approach to Modeling Multibody Systems with Compliant Moorings

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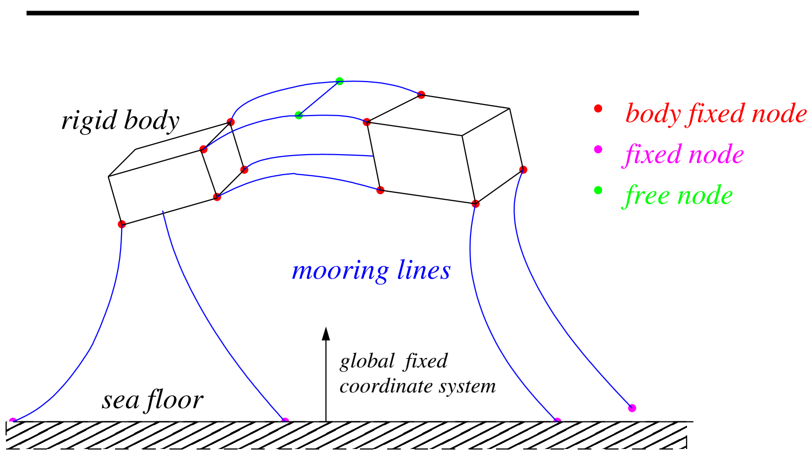
**Abstract**

There is considerable interest in the generation of electricity by harnessing the energy from waves and tidal streams [1]. The economic viability of these marine hydrokinetic (MHK) systems will depend on the ability to deploy such systems at low cost. In environments where the water is deep (greater than 50 m), it becomes necessary to consider compliant mooring (slack mooring) of the MHK devices. This paper details an approach to the modeling of networks of MHK devices that are secured via compliant moorings [2].

The model will be constructed using a Lagrangian dynamics framework. A lumped parameter discretization of the mooring lines will include the distributed effects of inertia, as well as hydrodynamic lift and drag forces. The mooring line model will allow for slackness, i.e, zero line tension, and account for the sea floor impenetrability constraints. In addition, the model construction will allow a network of mooring lines, and multiple rigid bodies attached to the lines. It will be shown that this modeling approach leads to a system of index-3 differential-algebraic equations (DAEs) with complementarity conditions.

Recently, Pecher et al. [2], and Fabien [4] presented static equilibrium analysis of compliant moored systems; while Raman-Nair and Baddour [5] analysed a single buoy secured with multiple lines. The analysis presented here considers the interaction multiple rigid bodies.

The main components of the system are: (1) rigid bodies; (2) mooring lines; and (3) body-fixed nodes, fixed nodes and free nodes used to define the network of mooring lines. A schematic of the of a typical model is shown below.




This model shows 2 rigid bodies connected to each other and the sea floor via a network of mooring lines. The drawing also shows body-fixed nodes, fixed nodes and free nodes.

**Kinematics** The kinematics of each rigid body is described using 3 translation degrees of freedom, and three rotation degrees of freedom (via Euler angles). The $i$th mooring line in the system is divided in to $N_i$ segments using $N_i+1$ nodes. Each node is described with 3 translation degrees of freedom. Thus, there are $3(N_i+1)$ degrees of freedom associated with the $i$th mooring line.

**Kinetics** The rigid bodies can have asymmetric inertial properties as well as added mass components. Moreover, the center of mass and the center of buoyancy need not be collocated. The mass of the line is distributed at the $N_i+1$ nodes and added mass of the line segments are also taken into account.

**Constraints** The connections between the mooring lines and rigid bodies give rise to equality constraints. The compliant mooring line condition gives rise to inequality constraints. Specifically, the compliant mooring line condition implies that the distance between adjacent nodes in the line can increase, but this distance can not be less than the undeformed length of the line segment. Thus, the mooring line will exhibit a chain-like behavior. The requirement that the mooring lines, and the rigid bodies, remain above the sea floor gives rise to additional inequality constraints.

**Applied Forces** The effects of the weight and buoyancy of each inertial element is accounted for in the dynamic equations of motion. Also included are the effects of lift and drag on the system elements due to motion relative to the fluid. Moreover, the fluid velocity is assumed to have spatial and temporal variation.

**Equations of Motion** Let $q(t) \in \mathscr{R}^n$ be the system displacements, $f(t) \in \mathscr{R}^n$ be the system velocities, $T(q,f,t)$ be the kinetic energy, $V(q)$ be the potential energy, $D(f)$ be the dissipation function, $\phi(q) \in \mathscr{R}^m$, $(m < n)$ be the equality constraints, $\psi(q) \in \mathscr{R}^p$ be the inequality constraints, and $e_s(t) \in \mathscr{R}^n$ be the applied forces. Then it can be shown that the equations of motion are given by a system of index-3 DAEs of the form [3]

$$
\begin{aligned}
0 &= \dot{q} - f, \\
0 &= M(q,f)\dot{f} + \Upsilon(q,f,t) + \phi_q^T \lambda + \psi_q^T \mu - e_s, \\
0 &= \phi(q), \\
0 &= \mu_i \psi_i(q),\ \mu_i \geq 0,\ \psi_i(q) \leq 0,\ i = 1,2,\cdots,p.
\end{aligned}
$$

Here, $M(q,f) = \partial^2 T/\partial f^2$, $\phi_q = \partial\phi/\partial q$, $\psi_q = \partial\psi/\partial q$, $\Upsilon = (\partial(\partial T/\partial f)/\partial q)f + \partial(\partial T/\partial f)/\partial t - \partial T/\partial q + \partial V/\partial q + \partial D/\partial f$. The variables $\lambda(t) \in \mathscr{R}^m$ and $\mu(t) \in \mathscr{R}^p$ are Lagrange multipliers associated with the equality and inequality constraints respectively. Note that the multipliers $\mu_i(t)$ are non-negative, and the complementarity conditions $\mu_i\psi_i(q) = 0$, $\mu_i \geq 0$, and $\psi_i(q) \leq 0$ makes these equations difficult to solve.

The paper also presents a numerical integration scheme to solve these DAEs using an interior-point method, which ensures that the inequality are satisfied at each time step.

## References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