# Computation of Two-Point Contact between Wheelset and Rail Exploiting the Structure of Non-Penetrating Contact Equations for Straight Track 

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#### Abstract

Computation of the contact points between wheel and rail is a fundamental problem in railroad vehicle modeling. In solving the governing equations for the contact points, two questions should be answered [1]. They are: 1) what is the number of contact points? And 2 ) where are their locations? To answer these questions, the non-penetrating contact equations are derived in track coordinate system [2]. It is shown that [3] five equations per contact should be constructed. Four of these equations locate common normal and the fifth equation makes the length of common normal to become zero. For two-contact-points problem [3], the Jacobean matrix per set of contact point can be partitioned as: $$
\mathrm{C}=\left[\begin{array}{ccc} c_{11} & 0 & c_{13} \\ c_{21} & 0 & c_{23} \\ c_{31} & c_{32} & c_{33} \end{array}\right]
$$ where, C is the Jacobean matrix and, the bloc matrices are: $$
\begin{array}{ll} \mathrm{c}_{11}=\left[\begin{array}{ccc} \frac{\partial \mathrm{f}_{1}}{\partial \alpha} & \frac{\partial \mathrm{f}_{1}}{\partial \mathrm{y}_{\mathrm{wp}}} & 0 \\ \frac{\partial \mathrm{f}_{2}}{\partial \alpha} & 0 & \frac{\partial \mathrm{f}_{2}}{\partial \mathrm{y}_{\mathrm{rp}}} \\ \frac{\partial \mathrm{f}_{4}}{\partial \alpha} & \frac{\partial \mathrm{f}_{4}}{\partial \mathrm{y}_{\mathrm{wp}}} & \frac{\partial \mathrm{f}_{4}}{\partial \mathrm{y}_{\mathrm{rp}}} \end{array}\right] & \mathrm{c}_{13}=\left[\begin{array}{cc} 0 & \frac{\partial \mathrm{f}_{1}}{\partial \varphi_{\mathrm{ctr}}^{\mathrm{ws}}} \\ 0 & \frac{\partial \mathrm{f}_{2}}{\partial \varphi_{\mathrm{ctr}}^{\mathrm{ws}}} \\ \frac{\partial \mathrm{f}_{4}}{\partial \mathrm{z}_{\mathrm{ctr}}^{\mathrm{ws}}} & \frac{\partial \mathrm{f}_{4}}{\partial \varphi_{\mathrm{ctr}}^{\mathrm{ws}}} \end{array}\right] \\ \mathrm{c}_{21}=\left[\begin{array}{lll} \frac{\partial \mathrm{g}}{\partial \alpha} & \frac{\partial \mathrm{~g}}{\partial \mathrm{y}_{\mathrm{wp}}} & \frac{\partial \mathrm{~g}}{\partial \mathrm{y}_{\mathrm{rp}}} \end{array}\right] & \mathrm{c}_{23}=\left[\begin{array}{ll} \frac{\partial \mathrm{g}}{\partial \mathrm{z}_{\mathrm{ctr}}^{\mathrm{ws}}} & \frac{\partial \mathrm{~g}}{\partial \varphi_{\mathrm{ctr}}^{\mathrm{ws}}} \end{array}\right] \end{array}
$$


Let $\Delta x$ and $V$ to be the vector of correction terms and functions whose zeros are sought. That is:

$$
\Delta x=\left[\Delta \alpha, \Delta y_{w p}, \Delta y_{r p}, \Delta s_{t r}, \Delta z_{c t r}^{w s}, \Delta \varphi_{c t r}^{w s}\right]^{T}, \quad V=\left[f_{1}, f_{2}, f_{4}, g, f_{3}\right]^{T}
$$

Partition $\Delta x$ and $V$ to match the partitions of the Jacobean matrix C. That is:

$$
\begin{gathered}
\Delta x=\left[\Delta x_{1}, \Delta x_{2}, \Delta x_{3}\right]^{T}, V=\left[V_{1}, V_{2}, V_{3}\right]^{T}, \text { where, } \Delta x_{1}=\left[\Delta \alpha, \Delta y_{w p}, \Delta y_{r p}\right]^{T}, \quad \Delta x_{2}=\Delta s_{t r}, \quad \Delta x_{3}= \\
{\left[\Delta z_{c t r}^{w s}, \Delta \varphi_{c t r}^{w s}\right]^{T} \text { and, } V_{1}=\left[f_{1}, f_{2}, f_{4}\right]^{T}, \quad V_{2}=g, \quad V_{3}=f_{3}}
\end{gathered}
$$

Then we have,

$$
\begin{gather*}
C_{11} \Delta x_{1}+C_{13} \Delta x_{3}=V_{1}  \tag{1}\\
C_{21} \Delta x_{1}+C_{23} \Delta x_{3}=V_{2}  \tag{2}\\
C_{31} \Delta x_{1}+C_{32} \Delta x_{2}+C_{33} \Delta x_{3}=V_{1} \tag{3}
\end{gather*}
$$

To compute the correction terms, use Equation (2) to solve for $\Delta x_{1}$ to get:

$$
\begin{equation*}
\Delta x_{1}=C_{11}^{-1} \cdot\left(V_{1}-C_{13} \cdot \Delta x_{3}\right) \tag{4}
\end{equation*}
$$

Substitute Equation (5) into Equation (3) to get:

$$
\begin{equation*}
\left[C_{23}-C_{21}\left(C_{11}\right)^{-1} C_{13}\right] \Delta x_{3}=V_{2}-C_{21}\left(C_{11}\right)^{-1} V_{1} \tag{5}
\end{equation*}
$$

Note that Equation (6) can be constructed for each contact point. Therefore, for two-contact-point problem, we have:

$$
\left[\begin{array}{l}
C_{23}^{1}-C_{21}^{1}\left(C_{11}^{1}\right)^{-1} C_{13}^{1}  \tag{6}\\
C_{23}^{2}-C_{21}^{2}\left(C_{11}^{2}\right)^{-1} C_{13}^{2}
\end{array}\right] \Delta x_{3}=\left[\begin{array}{l}
V_{2}^{1}-C_{21}^{1}\left(C_{11}^{1}\right)^{-1} V_{1}^{1} \\
V_{2}^{2}-C_{21}^{2}\left(C_{11}^{2}\right)^{-1} V_{1}^{2}
\end{array}\right]
$$

Note that the superscripts denote the contact point number. Equation (7) can be used to solve for $\Delta x_{3}$. Once $\Delta x_{3}$ is computed, use Equation (5) to compute $\Delta x_{2}$. To compute $\Delta x_{2}$, use Equation (4). That is:

$$
\begin{equation*}
C_{32} \Delta x_{2}=V_{3}-C_{31} \Delta x_{1}-C_{33} \Delta x_{3} \tag{7}
\end{equation*}
$$

Based on the method described, an algorithm is implemented to find the contact points. The results for one case are reported in Table 1. To verify the solution, the plot of the function F as described in [2] is reported in Figure 1. The zero of F is the $\mathrm{y}_{\mathrm{wp}}$ in Table 1. As can be seen the x -intercept of the graph matches the value of $y_{\mathrm{wp}}$ of the Table 1 .
Table 1: Contact points at initial guess of arc
length $0.2 \mathrm{~m}, \mathrm{Y}=0.01$ and $\psi=7^{\circ}$.

|  |  | Initial | Result |
| :---: | :---: | :---: | :---: |
| $\underset{\sim}{\Xi}$ | $\alpha$ | 179.822 | 179.946 |
|  | ywp | -0.007 | -0.004 |
|  | yrp | -0.004 | -0.001 |
|  | m | 0.11 | 0.109 |



Figure 1: Plot of left track at $\mathbf{y}=\mathbf{0} .01 \quad \phi=7^{\circ}$

In summary, the advantage of the method presented in this work is that, it uses block matrices whose dimensions are much smaller than the full Jacobean matrix. Hence it is more efficient than the method based on using the full Jacobean matrix.

## References

[1] Fallahi B. and Sunil K., 2010, "Joint International Conference on Multibody System Dynamics," The 1st Joint International Conference on Multibody System Dynamics, May 25-27, 2010, Lappeenranta, Finland
[2] Perla A. K., 2012, "Computation of Common Normals Between Wheel and Rail Surfaces," Master thesis, Northern Illinois University, Dekalb, IL
[3] C. Pan, 2014, "Computation of Contact between Wheelset and Rail Exploiting the Structure of Non-Penetrating Contact Equations for Straight Track", Master Thesis, Mechanical Engineering Department, Northern Illinois University

