Computation of Two-Point Contact between Wheelset and Rail Exploiting the Structure of Non-Penetrating Contact Equations for Straight Track

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Abstract

Computation of the contact points between wheel and rail is a fundamental problem in railroad vehicle modeling. In solving the governing equations for the contact points, two questions should be answered [1]. They are: 1) what is the number of contact points? And 2) where are their locations? To answer these questions, the non-penetrating contact equations are derived in track coordinate system [2]. It is shown that [3] five equations per contact should be constructed. Four of these equations locate common normal and the fifth equation makes the length of common normal to become zero. For two-contact-points problem [3], the Jacobean matrix per set of contact point can be partitioned as:

$$\mathbf{C} = \begin{bmatrix} c_{11} & 0 & c_{13} \\ c_{21} & 0 & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

where, C is the Jacobean matrix and, the bloc matrices are:

$$c_{11} = \begin{bmatrix} \frac{\partial f_1}{\partial \alpha} & \frac{\partial f_1}{\partial y_{wp}} & 0\\ \frac{\partial f_2}{\partial \alpha} & 0 & \frac{\partial f_2}{\partial y_{rp}}\\ \frac{\partial f_4}{\partial \alpha} & \frac{\partial f_4}{\partial y_{wp}} & \frac{\partial f_4}{\partial y_{rp}} \end{bmatrix} \qquad c_{13} = \begin{bmatrix} 0 & \frac{\partial f_1}{\partial \varphi_{ctr}^{ws}} \\ 0 & \frac{\partial f_2}{\partial \varphi_{ctr}^{ws}} \\ \frac{\partial f_4}{\partial z_{ctr}^{ws}} & \frac{\partial f_4}{\partial \varphi_{ctr}^{ws}} \end{bmatrix}$$
$$c_{21} = \begin{bmatrix} \frac{\partial g}{\partial \alpha} & \frac{\partial g}{\partial y_{wp}} & \frac{\partial g}{\partial y_{rp}} \end{bmatrix} \qquad c_{23} = \begin{bmatrix} \frac{\partial g}{\partial z_{ctr}^{ws}} & \frac{\partial g}{\partial \varphi_{ctr}^{ws}} \end{bmatrix}$$

Let Δx and V to be the vector of correction terms and functions whose zeros are sought. That is:

$$\Delta x = \begin{bmatrix} \Delta \alpha, \Delta y_{wp}, \Delta y_{rp}, \Delta s_{tr}, \Delta z_{ctr}^{ws}, \Delta \varphi_{ctr}^{ws} \end{bmatrix}^T, \qquad V = \begin{bmatrix} f_1, f_2, f_4, g, f_3 \end{bmatrix}^T$$

Partition Δx and V to match the partitions of the Jacobean matrix C. That is:

$$\Delta x = [\Delta x_{1,} \Delta x_{2}, \Delta x_{3}]^{T}, V = [V_{1}, V_{2}, V_{3}]^{T}, \text{ where, } \Delta x_{1} = [\Delta \alpha, \Delta y_{wp}, \Delta y_{rp}]^{T}, \quad \Delta x_{2} = \Delta s_{tr}, \quad \Delta x_{3} = [\Delta z_{ctr}^{ws}, \Delta \varphi_{ctr}^{ws}]^{T} \text{ and, } V_{1} = [f_{1}, f_{2}, f_{4}]^{T}, \quad V_{2} = g, \quad V_{3} = f_{3}$$

Then we have,

$$C_{11}\Delta x_1 + C_{13}\Delta x_3 = V_1 \tag{1}$$

$$C_{21}\Delta x_1 + C_{23}\Delta x_3 = V_2 \tag{2}$$

$$C_{31}\Delta x_1 + C_{32}\Delta x_2 + C_{33}\Delta x_3 = V_1 \tag{3}$$

To compute the correction terms, use Equation (2) to solve for Δx_1 to get:

$$\Delta x_1 = C_{11}^{-1} \cdot (V_1 - C_{13} \cdot \Delta x_3) \tag{4}$$

Substitute Equation (5) into Equation (3) to get:

$$[C_{23} - C_{21}(C_{11})^{-1}C_{13}]\Delta x_3 = V_2 - C_{21}(C_{11})^{-1}V_1$$
(5)

Note that Equation (6) can be constructed for each contact point. Therefore, for two-contact-point problem, we have:

$$\begin{bmatrix} C_{23}^1 - C_{21}^1 (C_{11}^1)^{-1} C_{13}^1 \\ C_{23}^2 - C_{21}^2 (C_{11}^2)^{-1} C_{13}^2 \end{bmatrix} \Delta x_3 = \begin{bmatrix} V_2^1 - C_{21}^1 (C_{11}^1)^{-1} V_1^1 \\ V_2^2 - C_{21}^2 (C_{11}^2)^{-1} V_1^2 \end{bmatrix}$$
(6)

Note that the superscripts denote the contact point number. Equation (7) can be used to solve for Δx_3 . Once Δx_3 is computed, use Equation (5) to compute Δx_2 . To compute Δx_2 , use Equation (4). That is:

$$C_{32}\Delta x_2 = V_3 - C_{31}\Delta x_1 - C_{33}\Delta x_3 \tag{7}$$

Based on the method described, an algorithm is implemented to find the contact points. The results for one case are reported in Table 1. To verify the solution, the plot of the function F as described in [2] is reported in Figure 1. The zero of F is the y_{wp} in Table 1. As can be seen the x-intercept of the graph matches the value of y_{wp} of the Table 1.



In summary, the advantage of the method presented in this work is that, it uses block matrices whose dimensions are much smaller than the full Jacobean matrix. Hence it is more efficient than the method based on using the full Jacobean matrix.

References

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