Nonlinear analysis of the cable-pulley interaction

Michal Hajžman*, **Radek Bulín***, **Pavel Polach***

* European Centre of Excellence, NTIS – New Technologies for Information Society
Faculty of Applied Sciences
University of West Bohemia
Univerzitní 8, 306 14 Plzeň, Czech Republic
mhajzman@kme.zcu.cz, [rbulin, ppolach]@ntis.zcu.cz

Abstract

The usage of cables, wires and fibres in various machines can be found in the real world and therefore it is necessary to have proper and effective modelling tools for sake of dynamic analysis and e.g. control synthesis. Absolute nodal coordinate formulation (ANCF) is capable to be used in the modelling of mechanical systems considering cables or wires as was demonstrated in [1] or [2]. This paper deals with a detailed analysis of the interaction between a rotating pulley (a sheave) and a cable, which is modelled using ANCF approach.

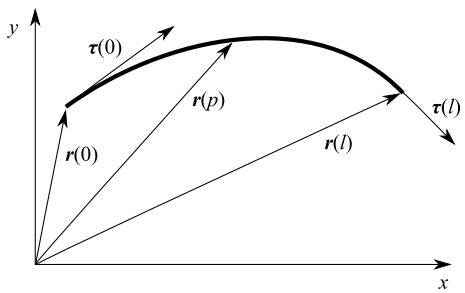

Figure 1: ANCF planar beam.

A planar ANCF beam element of length l with two nodes is employed (see Figure 1). The whole model of the ANCF planar beam element [3] is of the form

$$\mathbf{M}_e\ddot{\mathbf{e}} + \mathbf{K}_e(\mathbf{e})\mathbf{e} = \mathbf{Q}_{ek} \tag{1}$$

and is characterized by constant mass matrix $\mathbf{M}_e$, strongly nonlinear stiffness matrix $\mathbf{K}_e(\mathbf{e})$ derived using the strain energy and by vector of external forces $\mathbf{Q}_{ek}$. The assembling of a discretized flexible body (i.e. fibre, cable) model is straightforward and can be extended by a suitable model of viscous forces $\mathbf{B}(\dot{\mathbf{q}},\mathbf{q})\dot{\mathbf{q}}$

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}(\dot{\mathbf{q}},\mathbf{q})\dot{\mathbf{q}} + \mathbf{K}(\mathbf{q})\mathbf{q} = \mathbf{Q}_k, \tag{2}$$

where $\mathbf{q}$ is the vector of all elastic coordinates of the fibre. This model can be combined with the models of other flexible or rigid bodies and with the model of kinematic joints using a standard way.

In order to develop and verify the model of a cable-pulley interaction, at first the very simple system consisting of one pulley and one cable with defined forces on each side was studied. The more complex mechanical system (Figure 2) is additionally equipped by a linear drive (connected to a moving body using a fibre) and a pulley. This setup was also investigated experimentally [4] and therefore the simulation and experimental results could be compared.

The pulley is modelled as a rigid body with one degree of freedom (rotation), the fibre is modelled as deformable body and it is discretized using the ANCF method to n elements. During the interaction of these two bodies, the contact forces arise not only in element nodes. Therefore k equally spaced points are determined on each element. The contact forces are then evaluated for each of these point. Let us have element e ($e = 1, \ldots, n$) and point i ($i = 1, \ldots, k$). In each time step, the relative penetration δ_{ei}, penetration velocity $\dot{\delta}_{ei}$ and relative tangential velocity vector $\mathbf{v}_{ei}$ between the pulley and the fibre point is determined. Than the normal and friction forces are evaluated.

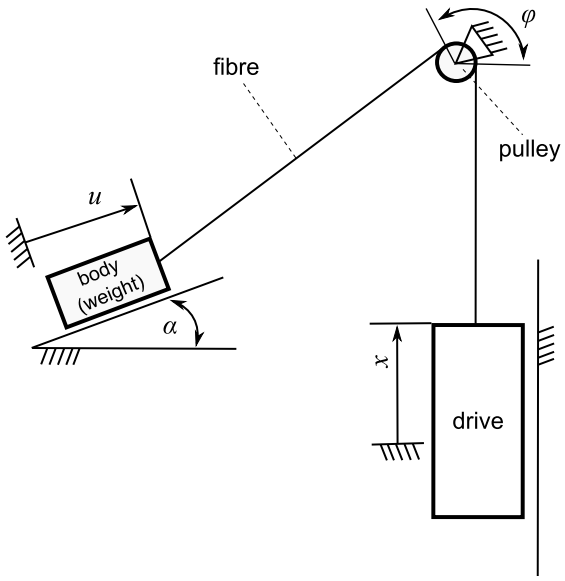

Figure 2: Scheme of the testing mechanical system composed of a motor, a pulley, a weight and a cable.

The best-known and very popular normal contact force formula was introduced by Hertz [5]. This model is based on the theory of elasticity and describes a force between two perfectly flexible solids with frictionless surfaces. The Hertz law can be expressed

$$F_{Nej} = K\delta_{ei}^{n}, \tag{3}$$

where K is the contact stiffness and n is the positive exponent, which reflects the shape of contact bodies. This basic contact force model does not consider the dissipation of energy during contact, therefore more complex models were developed [6]. Hunt-Crossley's model of normal contact force [5] was also tested as a more complex modelling approach. It is based on a simple dissipation model, where the damping coefficient is dependent on the penetration. The friction forces were also added to the interaction model and the model performance was tested with respect to various parameters of the friction model.

References

[1] H. Sugiyama, A.M. Mikkola, A.A. Shabana. A Non-Incremental Nonlinear Finite Element Solution for Cable Problems. Journal of Mechanical Design, Vol. 125, pp. 746–757, 2003.

[2] U. Lugrís, J.L. Escalona, D. Dopico, J. Cuadrado. Efficient and accurate simulation of the rope-sheave interaction in weight-lifting machines. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, Vol. 225, pp. 331–343, 2011.

[3] M. Berzeri, A.A Shabana. Development of simple models for elastic forces in the absolute nodal co-ordinate formulation. Journal of Sound and Vibration, Vol. 235, pp. 539–565, 2000.

[4] P. Polach, M. Hajžman, J. Václavík, Z. Šika, P. Svatoš. Model parameters influence of a simple mechanical system with fibre and pulley with respect to experimental measurements. In: Z. Terze, M. Vrdoljak (Eds.) Proceedings of ECCOMAS Thematic Conference Multibody Dynamics 2013, CD-ROM, Zagreb, Croatia, 2013.

[5] P. Flores, J. Ambrósio, J.C.P. Claro, H.M. Lankarani. Influence of the contact-impact force model on the dynamic response of multi-body systems. Proceedings of the IMechE, Part K: J. Multibody Dynamics, Vol. 220, pp. 21–34, 2006.

[6] M. Machado, P. Moreira, P. Flores, H.M. Lankarani. Compliant contact force models in multibody dynamics: Evolution of the Hertz contact theory. Mechanism and Machine Theory, Vol. 53, pp. 99–121, 2012.