New formulation of the rigid finite element method and its application to modelling of lines and risers

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Abstract

In problems of dynamics and control in offshore engineering long, slender elements undergoing large base motion often have to be considered. Those elements are subject to forces specific for the marine environment caused by waves and sea currents, and also hydrodynamic resistance and buoyancy forces. In order to discretise the slender elements usually the finite element, the finite segment and lumped mass methods are used. The rigid finite element method is also useful for those applications. The authors have worked out new formulations of the method, characterised by high numerical efficiency, which can be used in dynamic analysis and control of offshore structures [1].

The finite element method (FEM) [2] and the finite segment method [3,4] are used in commercial software packages such as RIFLEX and ORCAFLEX for modelling lines and risers. However, applicability of methods depends not only on their universality but also on numerical effectiveness, which involves both efficiency of the procedures of integrating the equations of motion and also the degree to which simplifications permissible from the engineer's point of view are possible; these simplifications determine the number of degrees of freedom of the system considered. A numerically effective method is the rigid finite element method (RFEM) [5], especially its new formulation, which enables shear, torsion or longitudinal stiffnesses to be eliminated without the necessity of reformulating the mathematical model of the system considered. The method of discretisation of a flexible link into rigid finite elements (rfe) and spring damping elements (sde) is shown in Figure 1.

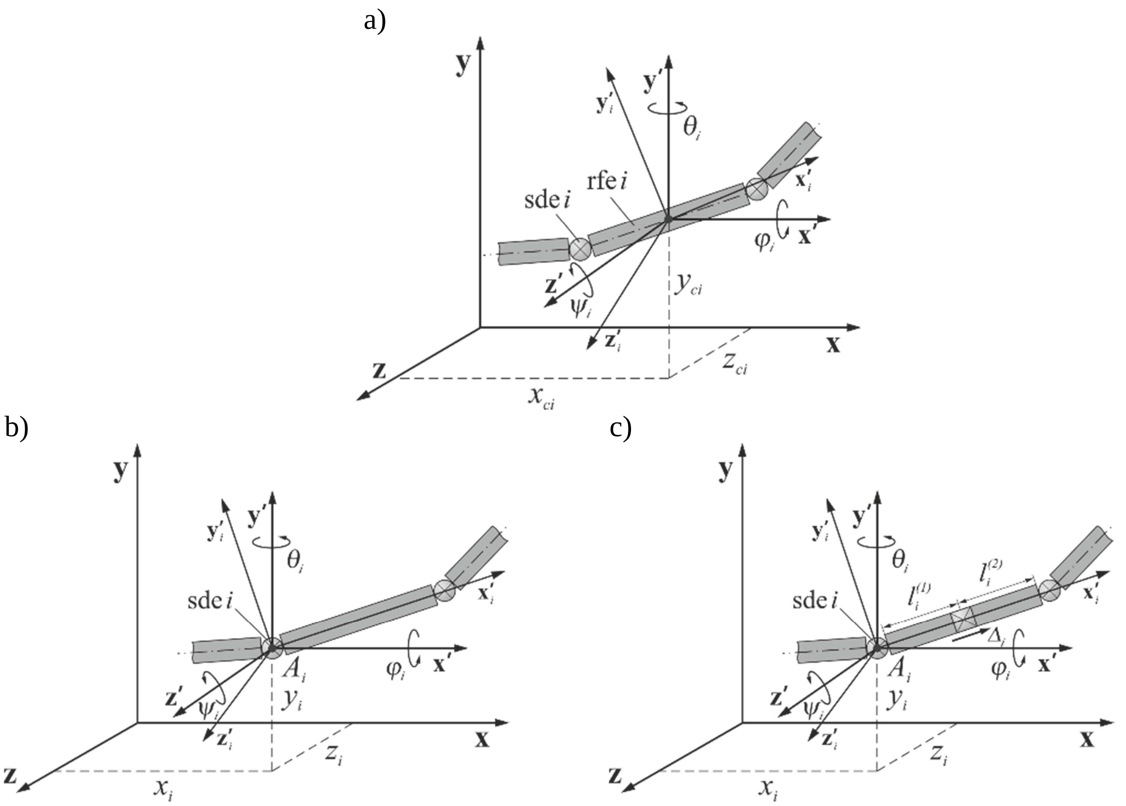

Figure 1: Discretisation of a flexible link

a) division into rfes and sdes in the classical formulation of the method
b) modified formulation

c) the approach proposed (new formulation)

In the classical formulation of the RFEM each rigid element has six degrees of freedom, and its generalised coordinates are the components of the vector:

$$q_i = \begin{bmatrix} x_i & y_i & z_i & \psi_i & \theta_i & \varphi_i \end{bmatrix}^T , \tag{1}$$

where: x_i, y_i, z_i are coordinates of the centre of the mass,

$\psi_i, \theta_i, \varphi_i$ are the ZYX Euler angles.

The rfes are connected by means of the sdes reflecting longitudinal, shear, bending and torsional stiffness [6]. Due to consideration of these stiffnesses, high frequencies occur in the analysis of free and forced vibrations, which requires small integration steps to be applied. When longitudinal and shear flexibilities are omitted, the vector of generalised coordinates can be written in the form:

$$q_i = \begin{bmatrix} \psi_i & \theta_i & \varphi_i \end{bmatrix}^T . \tag{2}$$

However, in the formulations of the method used up to now, such an approach results in the full mass matrix and thus lengthens the time of calculation.

In the new formulation we propose the vector of generalised coordinates in the following form:

$$q_i = \begin{bmatrix} x_i & y_i & z_i & \psi_i & \theta_i & \varphi_i & \Delta_i \end{bmatrix}^T , \tag{3}$$

where: Δ_i is the elongation of the element.

Rfes are connected by means of constraint equations, which eliminates large coefficients of the shear stiffness. Reactions at the connections are taken into account in the equations of motion. When Δ_i and/or φ_i in vector (3) are omitted, we obtain a model with bending flexibility, which is the flexibility that occurs most often in modelling lines and risers. The equations of motion are derived from the Lagrange equations using homogenous transformations. Numerous simulations which will be presented at the conference confirmed the high numerical efficiency and flexibility of this formulation of the rigid finite element method.

References

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