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A Kalman filter-based algorithm for IMU signals fusion applied to track geometry estimation

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Abstract
Ground vehicles undergo external excitations that come from path irregularities. The measurement of these irregularities is of great interest in many areas, as in the case of railway vehicle dynamics. Condition monitoring of track geometry with in-service vehicles by means of IMUs represents an interesting alternative because continuous operations are possible with a lower cost than with special recording vehicles, ([1]). In spite of these advantages, dealing with the noise and drift phenomenon of these sensors while fusing their data implies a challenging signal processing task.

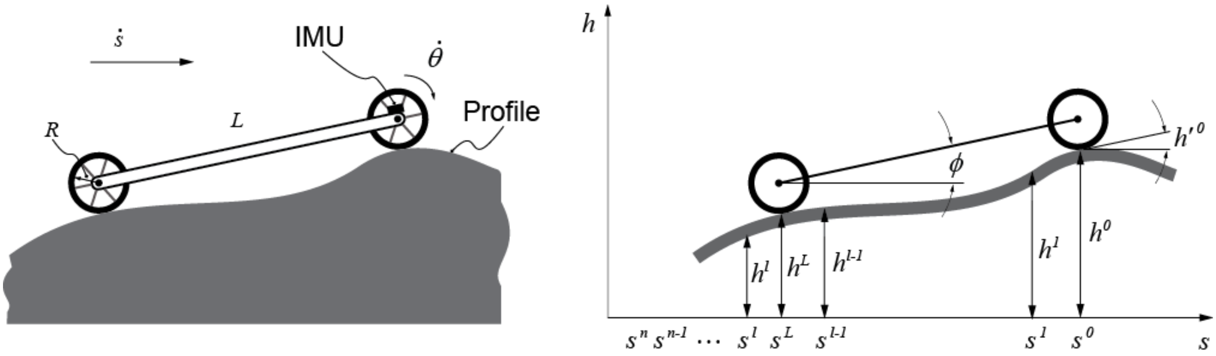

Figure 1: Monitoring vehicle

This work shows how to obtain the geometry of two-dimensional profiles based on IMU signals. Small curvatures are considered so, linearity conditions are accepted. Observing Figure 1, it is possible to obtain the relation between the slope h' and the vertical coordinate change rate $\dot{h}$ of the profile

$$\dot{h} = \dot{s}\,h' \tag{1}$$

On the other hand, the coordinate change rate of both wheels are related by the gyroscope signal as

$$\dot{\phi} = \left(\dot{h}^0 - \dot{h}^L\right)\Big/L \tag{2}$$

Based on the stochastic characteristic of track irregularities and in order to obtain an optimal estimate of its geometry, a Kalman filter algorithm [2] has been used to fuse IMU data signals. The discrete time evolution of a dynamic system where *k* indicates the time index can be described as follows

$$\mathbf{x}_{k+1} = \mathbf{A}_k\mathbf{x}_k + \mathbf{N}_k\mathbf{u}_k + \mathbf{w}_k \tag{3}$$

$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k \tag{4}$$

where **x** is the system state vector, **u** is a control input, **y** is the measurement vector and **w** and **v** represent the noises. **A**, **C** and **N** are matrices with appropriate dimensions. The true value of the acceleration at point 0 can be decomposed by the accelerometer measurement, its bias and noise as $\ddot{h} = \ddot{h}^{measured} + \ddot{h}^{bias} + w^{\ddot{h}}$. Thus, the vertical coordinate and of the front wheel at time $k+1$ can be expressed as follows

$$h_{k+1}^{0} = h_{k}^{0} + \dot{h}_{k}^{0}\Delta t_{s} + \left(1/2\right)\Delta t_{s}{}^{2}\left(\ddot{h}_{k}^{measured} + \ddot{h}_{k}^{bias} + w_{k}^{\ddot{h}}\right) \tag{5}$$

Equation (5) indicates that if h_k^0, $\dot{h}_k^0$ and $\ddot{h}_k^{bias}$ are considered as state variables, the measured acceleration can be considered as a control variable while $w_k^{\ddot{h}}$ is white noise. The sampling rate Δt_s is a known parameter based on the shortest wave length irregularities of interest and the fastest velocity to be reached. The velocity at point 0 will be $\dot{h}_{k+1}^{0} = \dot{h}_{k}^{0} + \Delta t_{s}\left(\ddot{h}_{k}^{measured} + \ddot{h}_{k}^{bias} + w_{k}^{\ddot{h}}\right)$.

As it can be seen in Equation (2), the angular velocity is directly related to the velocities of the end points of the vehicle. As the time interval Δt_s will usually be shorter than the time that the rear wheel takes to reach the same position that the front wheel, so many samplings will occur in this time. So, in order to use Equation (2), a relation between the coordinate of the end points is required. This relation can be easily established considering that as the vehicle advances, the coordinate of the point 1 at time $k+1$ will be the coordinate that point 0 had at the previous time k

$$\begin{aligned} &h_{k+1}^{1} = h_{k}^{0}; \quad \ldots \quad ; h_{k+1}^{i+1} = h_{k}^{i}; \qquad i = 0, \ldots, n-1 \\ &s_{k+1}^{1} = s_{k}^{0} \quad \ldots \quad ; s_{k+1}^{i+1} = s_{k}^{i}; \qquad i = 0, \ldots, n-1 \end{aligned} \tag{6}$$

Extending this reasoning to all the intermediate points between the end points, the velocity of point L can be obtained as $\dot{h}_k^L = \dot{s}_k^0 h_k'^L$,where $h_{k+1}'^L$ is obtained from the following interpolation

$$h_k'^L = \left(h_k^l - h_k^{l-1}\right) / \left(s_k^l - s_k^{l-1}\right) \tag{7}$$

where the horizontal coordinate is obtained as $s_{k+1}^{0} = s_{k}^{0} + \dot{s}_{k}^{0}\Delta t + \left(1/2\right)\left(\ddot{s}_{k}^{measured} + \ddot{s}_{k}^{bias} + w_{k}^{\ddot{s}}\right)\Delta t^{2}$ and $l < n$ in accordance with slow velocities. In the same way, the horizontal velocity is

$$\dot{s}_{k+1}^{0} = \dot{s}_{k}^{0} + \Delta t_{s}\left(\ddot{s}_{k}^{measured} + \ddot{s}_{k}^{bias} + w_{k}^{\ddot{s}}\right) \tag{8}$$

The previous analysis suggests that not only must the coordinate and velocity of the end point 0 be state variables, but also the coordinate of all those between the intermediate points as well, so

$$\mathbf{x}_k = \left[h_k^0 \;\; s_k^0 \;\; \dot{h}_k^0 \;\; \dot{s}_k^0 \;\; h_k^1 \;\; s_k^1 \;\; h_k^2 \;\; s_k^2 \;\; \ldots \;\; h_k^n \;\; s_k^n \;\; \ddot{h}^{bias} \;\; \ddot{s}^{bias}\right]^T \tag{9}$$

Additionally to the encoder and gyroscope signals, our measurement model introduces a virtual displacement sensor in order to render the system observable. If this virtual sensor is supposed to be always measuring zero, the error of this equation will be the magnitude of the irregularity. In order to use the encoder signal as a measurement variable, it is supposed that its value is zero at s_0^0, i.e. at s_k^0 for $k = 0$

$$\theta_k = \left(s_k^0 - s_0^0\right) / R + \dot{\theta}_k \tag{10}$$

In this way, using the virtual sensor, the gyroscope signal represented by Equation (2) and the encoder expression of Equation (10), the measurement vector will be

$$\mathbf{y}_k = \left[0 \;\; \dot{\phi}_k \;\; \theta_k\right]^T \tag{11}$$

The transition and measurement matrices **A** and **C** can be easily obtained from Equations previously deduced.

References

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[2] M.S. Grewal, and A.P. Andrews. Kalman Filtering: Theory and Practice Using MATLAB. Wiley, Hoboken, N.J. 2011.