Modeling of multilayer beams using the Absolute Nodal Coordinate Formulation

Grzegorz Orzechowski, Janusz Frączek

Institute of Aeronautics and Applied Mechanics
Warsaw University of Technology
Nowowiejska 24, 00-665 Warsaw, Poland
gorzech@meil.pw.edu.pl, jfraczek@meil.pw.edu.pl

Abstract
Absolute nodal coordinate formulation (ANCF) [1] is the method for dynamic analysis of the flexible multibody systems that undergo large displacements, rotations and deformations. One of the important feature of the ANCF is description consistent with the general continuum mechanics theory. This paper presents results of the case study of modeling multilayer structures with fully parametrized ANCF elements. This type of structures is often use to model composite materials in aerospace, automotive and ship vehicles [2]. In the following considerations a standard fully parametrized ANCF planar beam element [3] is used.

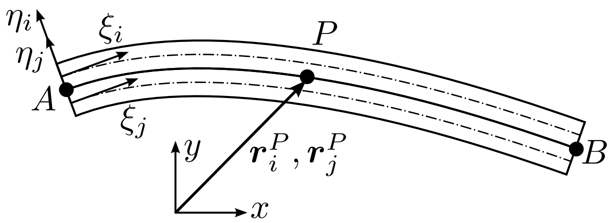

Figure 1: Two-layer beam structure.

Figure 1 presents a two-layer flexible body consists of two joined planar beam elements. In order to create exact connection between elements, their position vectors have to be equal at each point P on the common boundary, which may be written as:

$$\boldsymbol{r}_i^P = \boldsymbol{r}_j^P \tag{1}$$

where $\boldsymbol{r}_i^P$ and $\boldsymbol{r}_j^P$ are position vectors of the point P on the elements, respectively, i and j. The element i position vector can be expressed in terms of the element coordinates $\boldsymbol{e}_i$ as $\boldsymbol{r}_i = \boldsymbol{S}\boldsymbol{e}_i$ where $\boldsymbol{S}(\xi_i, \eta_i)$ is the element shape functions matrix [3] that depends on element dimensionless coordinates $\xi_i = x_i/l_i$ and $\eta_i = y_i/l_i$ for l_i being the element length. It should be noted that when both elements have equal lengths, values of coordinates ξ_i and ξ_j are equal for the given point P. Therefore one can rewrite equation (1) into more appropriate form (for elements lengths $l_i = l_j = l$):

$$\boldsymbol{S}\left(\xi, \eta_i^{\min}\right)\boldsymbol{e}_i = \boldsymbol{S}\left(\xi, \eta_j^{\max}\right)\boldsymbol{e}_j \tag{2}$$

where the condition $\xi_i = \xi_j = \xi$ is used and the constant values of the lateral dimensionless coordinates are $\eta_i^{\min} = -h_i/(2l)$ and $\eta_j^{\max} = h_j/(2l)$, while h_i and h_j are elements heights. This equation has to be satisfied for all values of ξ.
The planar beam element shape functions are cubic along the beam center-line and linear in transverse direction, therefore to join elements exactly, four conditions are required in longitudinal direction and two in transverse direction. Those conditions may be written as follows:

$$\begin{cases} \boldsymbol{S}\left(\xi_A, \eta_i^{\min}\right)\boldsymbol{e}_i = \boldsymbol{S}\left(\xi_A, \eta_j^{\max}\right)\boldsymbol{e}_j, & \boldsymbol{S}\left(\xi_B, \eta_i^{\min}\right)\boldsymbol{e}_i = \boldsymbol{S}\left(\xi_B, \eta_j^{\max}\right)\boldsymbol{e}_j, \\ \boldsymbol{S}_{,x}\left(\xi_A, \eta_i^{\min}\right)\boldsymbol{e}_i = \boldsymbol{S}_{,x}\left(\xi_A, \eta_j^{\max}\right)\boldsymbol{e}_j, & \boldsymbol{S}_{,x}\left(\xi_B, \eta_i^{\min}\right)\boldsymbol{e}_i = \boldsymbol{S}_{,x}\left(\xi_B, \eta_j^{\max}\right)\boldsymbol{e}_j, \\ \boldsymbol{S}_{,y}\left(\xi_A, \eta_i^{\min}\right)\boldsymbol{e}_i = \boldsymbol{S}_{,y}\left(\xi_A, \eta_j^{\max}\right)\boldsymbol{e}_j, & \boldsymbol{S}_{,y}\left(\xi_B, \eta_i^{\min}\right)\boldsymbol{e}_i = \boldsymbol{S}_{,y}\left(\xi_B, \eta_j^{\max}\right)\boldsymbol{e}_j. \end{cases} \tag{3}$$

where $\xi_A = 0$, so $\left(\xi_A, \eta_i^{\min}\right)$ and $\left(\xi_A, \eta_j^{\max}\right)$ indicates the point A in Figure 1 while $\xi_B = 1$, so $\left(\xi_B, \eta_i^{\min}\right)$ and $\left(\xi_B, \eta_j^{\max}\right)$ indicates B. Moreover, $\boldsymbol{S}_{,x}$ and $\boldsymbol{S}_{,y}$ are the shape functions matrix partial derivatives with respect to x and y. One can notice, that the choice of conditions (3) is not unique. Moreover, it can be shown that the substitution of conditions (3) into Equation (2) results in an identity. Therefore, the set of conditions (3) allow to create a two-layer structure consists of two ANCF fully parametrized planar beams with exact continuity of the displacement field. This procedure may be applied many times to create a multilayer structure with arbitrary number of elements.

The set of Equations (3) provides twelve linear constraint equations that can be eliminated in the preprocessing stage. It means that the body with elements arranged in layers have the same number of degrees of freedom like the system with one layer. This is an important feature of presented approach, because the number of the coordinates that are integrated is the same irrespective of the number of layers.

When the body is composed with many elements, which is the case in most practical applications, Equations (3) should be applied to each pair of adjoined elements i and j that form the multilayer section. However, connectivity conditions between elements and constraints that connect flexible body with the multibody system, should be applied only to the single layer to avoid constraints redundancy.

The same procedure may be use to create a multilayer model with other fully parametrized beam or plate ANCF elements. For example the spatial ANCF beam introduced in [4, 5] has shape functions that are cubic in longitudinal direction and linear in transverse directions. Therefore, it is sufficient to add two vector conditions to Equations (3) for the gradient along z in the form $\boldsymbol{r}_{i,z}^A = \boldsymbol{r}_{j,z}^A$ and $\boldsymbol{r}_{i,z}^B = \boldsymbol{r}_{j,z}^B$.

Presented approach was tested with several numerical examples of modal and static analysis and the good agreement with reference results was observed. Moreover, when all elements in the structure have the same material properties, the multilayer model is reduced to the single layer model with combined height. For example the modal analysis of the simply supported beam, like one presented in Section 4.2.2 in Gerstmayr et al [6], shows that frequencies for given modes agree very well for two-layer and single layer models.

Acknowledgments

This work was supported by Polish National Science Centre (Decision No. DEC-2012/07/B/ST8/03993).

References

[1] A.A. Shabana. Definition of the slopes and the finite element absolute nodal coordinate formulation. Multibody System Dynamics, Vol. 1, No. 3, pp. 339–348, 1997.

[2] E. Carrera. Theories and finite elements for multilayered, anisotropic, composite plates and shells. Archives of Computational Methods in Engineering, Vol. 9, No. 2, pp. 87–140, 2002.

[3] M. Omar, A.A. Shabana. A two-dimensional shear deformable beam for large rotation and deformation problems. Journal of Sound and Vibration, Vol. 243, No. 3, pp. 565 – 576, 2001.

[4] A.A. Shabana, R.Y. Yakoub. Three dimensional absolute nodal coordinate formulation for beam elements: Theory. Journal of Mechanical Design, Vol. 123, No. 4, pp. 606–613, 2001.

[5] R.Y. Yakoub, A.A. Shabana. Three dimensional absolute nodal coordinate formulation for beam elements: Implementation and applications. Journal of Mechanical Design, Vol. 123, No. 4, pp. 614–621, 2001.

[6] J. Gerstmayr, M.K. Matikainen, A.M. Mikkola. A geometrically exact beam element based on the absolute nodal coordinate formulation. Multibody System Dynamics, Vol. 20, No. 4, pp. 359–384, 2008.