# A Complementarity Problem Approach for Multibody Contact Dynamics with Regularized Friction

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## Abstract

Contact modelling, simulation, and analysis of mechanical systems with friction need proper treatments due to the non-smooth nature of the contact phenomenon. Two main approaches to deal with the contact problems are constraint and compliant based formulations [3]. In constraint based approaches, the problem is formulated by imposing the kinematic unilateral constraints between bodies in contact. This forms a complementarity condition, meaning that either normal contact velocity or force exists at each time instance. By adding the Coulomb friction, this phenomenon can be represented by a complementarity problem with a friction cone. To solve the resulting complementarity problem, the actual cone may be tackled via appropriate iterative techniques [2]. Alternatively, the friction cone can be linearized [1]. In the latter case, various available pivoting or iterative algorithms can be applied to solve the resulting linear complementarity problem (LCP) [4]. On the other hand, in the compliant based approaches, the complementarity conditions disappear by relaxing the normal direction and characterizing the normal force based on the stiffness and normal penetration. A selected friction model is then incorporated into the equations. This leads us to a set of differential equations in which appropriate numerical techniques should be used to solve the equations. While selecting high stiffness values increases the accuracy of the model, it can imply a set of stiff differential equations which encompasses challenges and drawbacks.

A novel complementarity problem formulation based on the combination of the above approaches is introduced in this paper. In this approach, contacts are characterized based on the kinematic constraints while the friction forces are simultaneously regularized and incorporated into the formulation. As a result, the mathematical formulation representing the model is a complementarity problem whose variables are the normal forces. The friction forces are eliminated because of their dependencies to the normal forces, positions, or velocities. More specifically, the friction forces are expressed in terms of the normal forces in the kinetic phase, and they depend on the positions or velocities in the static phase (depending on the selected friction model). It can be shown that if the equations of motion are discretized, the resulting LCP in our proposed formulation reads as

$$\begin{bmatrix} \mathbf{M} & -\mathbf{J}^T & -(\mathbf{D}+\mathbf{D_k})^T \\ \mathbf{J} & \mathbf{0} & \mathbf{0} \\ (\mathbf{D}+\mathbf{D_k}) & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \boldsymbol{\lambda} \\ \boldsymbol{\gamma}_n \end{bmatrix} + (\mathbf{b}+\mathbf{b_s}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{w} \end{bmatrix}, \quad \mathbf{0} \leq \boldsymbol{\gamma}_n \perp \mathbf{w} \geq \mathbf{0} \tag{1}$$

where $\mathbf{M}$ is the generalized mass matrix, $\mathbf{J}$ is the bilateral Jacobian matrix, $\mathbf{D}$ is the Jacobian matrix of the unilateral constraints in the normal direction, $\mathbf{D_k}$ is the additional term appears due to the dependency of the kinetic friction impulses to the normal impulses, $\mathbf{v}$ is the generalized velocities, $\boldsymbol{\lambda}$ is the vector of bilateral constraint impulses, $\boldsymbol{\gamma}_n$ is the vector of unilateral constraint impulses in the normal direction, $\mathbf{w}$ is the vector of normal contact velocities, $\mathbf{b}$ is the vector containing remaining terms of the complementarity formulation including those related to the equations of motion, and $\mathbf{b}_s$ is the additional term introduced as a result of the static friction regularization with respect to the position or velocity.

The velocity of each contact point is compared to a pre-defined velocity threshold to decide whether the point is in the static or the kinetic phase. For each contact point at each time step, the friction phase is determined relying on the velocities of the previous time step. In this paper, appropriate formulations will be derived for $\mathbf{D_k}$ and $\mathbf{b}_s$. For each contact point at each time step, $\mathbf{D_k}$ appears in the formulation if the point experiences the kinetic phase otherwise vanishes. On the contrary, $\mathbf{b}_s$ appears in the formulation if the point is in the static phase. It is remarkable that Eq. 1 can be reduced to a lower dimensional LCP

whose variables are solely unilateral normal impulses. Velocity values and bilateral impulses are then obtained through a closed formula by using the LCP solution. The resulting LCP can be solved via the available well-established numerical techniques [4].

This general representation can incorporate different friction models in our setting, i.e. models based on the position regularization such as Bristle, LuGre, etc., or the velocity regularization [5]. With this novel representation, the resulting LCP's dimension is reduced. Therefore, the computational cost of the proposed approach is less expensive, which could be of great importance, specially, for real-time simulation of mechanical systems with large number of contacts. Furthermore, in this approach, the challenges of handling the stiff differential equations in the pure compliant based approaches will be avoided as no relaxation is imposed in the normal direction. The applicability of the proposed formulation will be illustrated and further validated for number of benchmark examples.

**References**

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