Interpolation-based Parametric Model Order Reduction for Elastic Multibody Systems

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Abstract

The consideration of elastic effects as well as nonlinear rigid body motions makes the description of mechanical systems by elastic multibody systems (EMBS) a powerful tool in the development process. Due to the fine spatial discretization of the elastic continuum, the reduction of the elastic degrees of freedom is necessary. This essential step to enable efficient EMBS simulations can be executed by various model reduction methods. Reduction techniques developed in advanced mathematics [1] are extended and applied for elastic multibody systems [2, 3]. The quality of the reduced systems highly depends on the approximation of the input-output behavior which is determined for inputs and outputs which represent the acting forces and the nodal deformation of interest.

For an increasing number of applications, e.g. simulation of turning and milling processes, gear-wheels, sliding components, and cranes, the acting forces can vary their position onto the elastic body. To consider all possibly actuated nodes in the reduction process as inputs, very large input matrices occur which aggravate the determination of small reduced elastic systems. Due to the fact that for many mechanical examples only a small number of nodes is actuated simultaneously, here, the input matrix is described by a parameter dependent combination of local input matrices

$$\boldsymbol{B}(p) = \sum_{i=1}^{k} \omega_i(p)\boldsymbol{B}_i. \tag{1}$$

Thereby, the parameter p describes the position of the acting forces and the weighting function ω is used for the summation of all local support matrices $\boldsymbol{B}_i$.

For the model reduction step, the parameter dependent linear time-invariant differential equation

$$\begin{aligned} \boldsymbol{M}\cdot\ddot{\boldsymbol{q}}(t)+\boldsymbol{D}\cdot\dot{\boldsymbol{q}}(t)+\boldsymbol{K}\cdot\boldsymbol{q}(t) &= \boldsymbol{B}(p)\cdot\boldsymbol{u}(t), \\ \boldsymbol{y}(t) &= \boldsymbol{C}\cdot\boldsymbol{q}(t) \end{aligned} \tag{2}$$

should be reduced while retaining the parameter dependency. Equation (2) contains the mass matrix $\boldsymbol{M}$, damping matrix $\boldsymbol{D}$ and stiffness matrix $\boldsymbol{K}$ which describe the internal dynamic of the elastic body, the elastic degrees of freedom $\boldsymbol{q}$, the input $\boldsymbol{u}$ which is mapped by the parameter dependent input matrix $\boldsymbol{B}(p)$, and the outputs $\boldsymbol{y}$ which represent the elastic deformation of certain nodes determined by the output matrix $\boldsymbol{C}$. The reduction is performed by a Petrov-Galerkin projection which approximates the original degrees of freedom $\boldsymbol{q}$ by a smaller number of reduced elastic degrees of freedom $\bar{\boldsymbol{q}}$ with $\boldsymbol{q}\approx\boldsymbol{V}\cdot\bar{\boldsymbol{q}}$.

In recent years, parametric model order reduction (PMOR) methods have been developed to reduce systems similar to Equation (2), see [4] for a good overview. Parametric model reduction methods are often divided in global and local approaches. In global PMOR only one representative projection matrix $\boldsymbol{V}$ is determined by combining projection matrices for different sampling points. In contrast, the local PMOR methods determine a new reduced system by interpolation of locally reduced support systems. In [5, 6] the local PMOR methods based on matrix interpolation are extended for second order mechanical systems and investigated in frequency domain as well as in time simulations of moving loads. In this contribution, two other methods based on interpolation are developed and analyzed for mechanical systems. Apart from interpolating the reduced system matrices, the interpolation of the projection matrices $\boldsymbol{V}_i$ or the interpolation of the transfer function are possible. The different interpolation techniques are analyzed regarding their applicability for elastic multibody systems and their online-offline-decomposition. Results in frequency domain are shown for the comparison between the different techniques, see Figure 1 for the relative error of the different techniques applied for a thin-walled cylinder example.

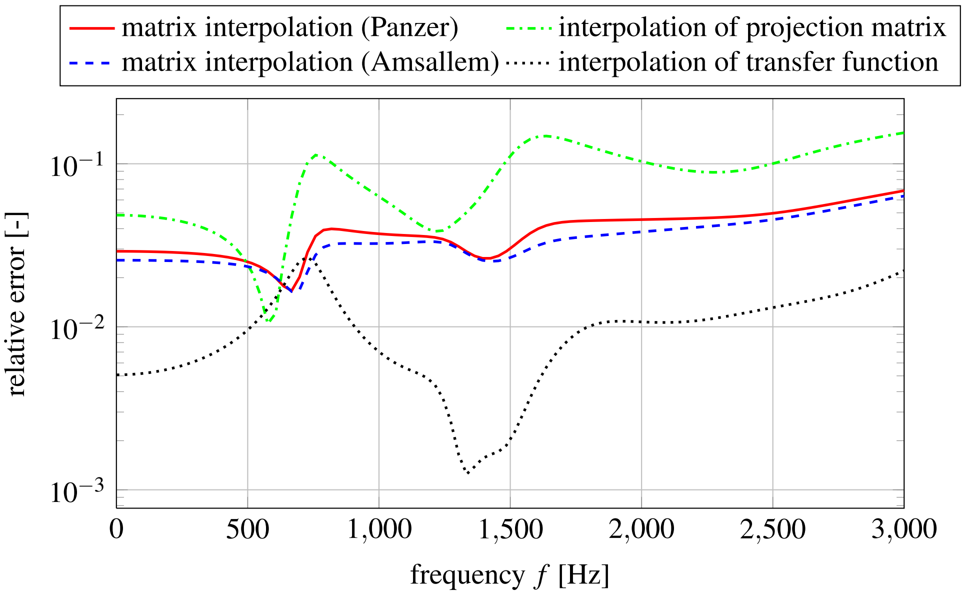

Figure 1: Comparison of different interpolation variations for parameter dependent elastic bodies.

References

[1] Antoulas, A.: Approximation of Large-Scale Dynamical Systems. Philadelphia: SIAM, 2005.

[2] Lehner, M.: Modellreduktion in elastischen Mehrkörpersystemen (in German), Vol. 10 of Dissertation, Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart. Aachen: Shaker Verlag, 2007.

[3] Fehr, J.: Automated and Error-Controlled Model Reduction in Elastic Multibody Systems, Vol. 21 of Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart. Aachen: Shaker Verlag, 2011.

[4] Benner, P.; Gugercin, S.; Willcox, K.: A Survey of Model Reduction Methods for Parametric Systems. Max Planck Institute Magdeburg Preprint MPIMD13-14, 2013.

[5] Fischer, M.; Eberhard, P.: Simulation of Moving Loads in Elastic Multibody Systems with Parametric Model Reduction Techniques. The Archive of Mechanical Engineering, Vol. 61, No. 2, pp. 209–226, 2014.

[6] Fischer, M.; Eberhard, P.: Application of Parametric Model Reduction with Matrix Interpolation for Simulation of Moving Loads in Elastic Multibody Systems. Advances in Computational Mathematics, 2014. DOI 10.1007/s10444-014-9379-7