On the choice of coordinates for flexible multibody systems

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Abstract

Currently, there are several ways to model flexible multibody systems and, in extension to that, there are even more ways to numerically simulate flexible multibody dynamics systems.

The first and direct approach, see Figure 1c, for the modeling of flexible bodies which undergo large rigid body motions and small deformations, is to use (total) displacements measured in an inertial frame within a continuum mechanics formulation for the virtual work of elastic forces, of inertia force and of external forces according to D'Alembert's principle in Lagrange's form. The work of elastic forces needs to be formulated by means of a geometrically nonlinear strain measure. This results in a strain energy that is unaffected by the underlying rigid body motion, which would not be possible by a geometrically linear formulation indicated in Figure 1a. Using the simplest possible strain measure, such as the fully nonlinear Green Lagrange strain, the numerical simulation is time consuming due to a non-constant stiffness matrix, which needs to be updated frequently. This is caused by the strong coupling between shearing or compression and the rotation of the flexible bodies. As an advantage, the formulation of the kinetic energy and – as a consequence – the computation of inertia terms in a finite element description, is simple and efficient. Finally, the direct approach offers no way to reduce the size of the discretized system of equations of motion in an efficient way, up to now.

The second and more traditional approach, also denoted as floating frame of reference, is based on relative displacements, see Figure 1b. The total displacements are split into a rigid body motion and a superimposed (usually small) deformation. The rigid body motion is commonly given as translation plus a rigid body rotation, which is described with rotational parameters. Regarding the work of elastic forces, the floating frame of reference formulation corresponds to a co-rotational formulation, which is sometimes used in static finite element methods (FEM) for large deformation problems. Due to the specific assumption that deformations within each body are small (i.e. small deformation gradients) and due to a co-rotational formulation of flexible deformation, the co-rotational formulation leads to a constant stiffness matrix within a finite element formulation. Unfortunately, the inertia terms follow from the time-wise differentiation of the flexible displacements superimposed to rigid body motion, which leads to undesirably nonlinear inertia terms.

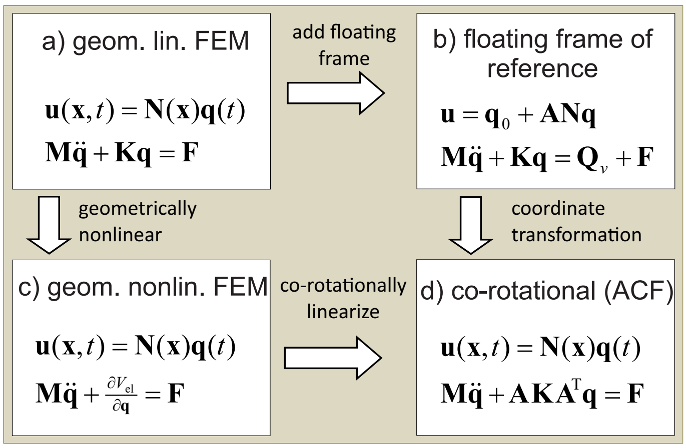

Figure 1: An overview of different FEM based formulations for flexible multibody dynamics.

Typical finite elements, which provide a linear configuration space for the deformed body The most common finite elements, which are popular due to the versatility of application and the comparatively simple implementation, are solid finite elements. Typically, the interpolation of nodal displacements $\mathbf{u}$ is given by a summation over shape functions $\mathbf{N}_i$ and elastic coordinates q_i

$$\mathbf{u} = \sum_i \mathbf{N}_i q_i \tag{1}$$

As an example, a planar triangular finite element with linear interpolation functions can be mentioned. This triangular finite element has 6 degrees of freedom. If properly implemented, this element can undergo rigid body motion, which corresponds to 3 degrees of freedom for translation and rotation, and inplane deformation, which corresponds to 3 degrees of freedom for stretch and shear deformation. Obviously, if a flexible (multi)body – which undergoes only small deformations – is modeled by means of triangular finite elements, the rigid body motion is almost the same for all bodies. As a second example, the absolute nodal coordinate (ANC) formulation is mentioned, which has been developed for the modeling and simulation of structural elements of flexible multibody systems. The mapping of nodal coordinates to global displacements is, again, linear in case of ANC finite elements and follow relation (1). Due to the isoparametric interpolation of displacements, any rigid body motion can be represented with ANC finite elements.

In order to combine the direct approach and the floating frame of reference formulation, a co-rotational formulation which uses the coordinates of the direct approach has been introduced [1], see Figure 1d. The first step towards an efficient formulation is based on a co-rotational strain tensor, which is given as the co-rotationally linearized Green Lagrange strain $\tilde{\mathbf{E}}$, using the underlying rigid body rotation $\mathbf{A}$,

$$\tilde{\mathbf{E}} = \mathbf{A}^T \nabla \mathbf{u}_F \tag{2}$$

In this way, a finite element formulation of the flexible multibody system leads to a constant mass matrix and a co-rotationally linearized stiffness matrix - also denoted as absolute coordinate formulation, see Gerstmayr and Schöberl [1]. The computational efforts for each time step are much smaller as compared to a fully geometrically nonlinear formulation. In addition to that, the absolute coordinate formulation offers

1. a method that is dual to the floating frame of reference formulation, and the relation between both methods can be interpreted as a coordinate transformation
2. good numerical properties due to the linear configuration space, thus energy-momentum conservation can be obtained with little modification of the time integration method [3]

In order to obtain an efficient flexible multibody dynamics formulation, the linear configuration space – which may be large – can be tremendously reduced, similar to standard model order reduction methods in multibody systems [2]. The model order reduction is a requirement for the simulation of realistic spatial multibody systems modeled by large scale finite element models. The reduction method preserves the linear configuration space. Thus, the application of energy-momentum preserving integration schemes [3] is straight forward and the extension to large deformations is feasible. This paper provides a framework of co-rotational formulations, shows the duality of the floating frame of reference and co-rotational formulation with inertial (absolute) coordinates and shows potential extensions.

References

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