Multibody Kinematics. A Topological Formulation Based on Structural-Group Coordinates.

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Abstract

Kinematic analysis plays a fundamental role in multibody dynamics. Moreover it is frequently used in synthesis problems, as a first stage in the design of mechanical systems and, in other cases, the interest in the multibody system is purely kinematic (position analysis, range of movement, etc.). To perform a kinematic analysis, the configuration of the multibody system has to be described using a specific set of coordinates $\boldsymbol{q}$: topological formulations use relative coordinates and global formulations use Cartesian or natural coordinates. When closed kinematic chains are present, the number of coordinates that describe the system is larger than its mobility, so it becomes necessary to define a vector $\boldsymbol{\Phi}(\mathbf{q},t)$ of constraint equations that relate the $\boldsymbol{q}$ coordinates to each other (1). Once this system of equations is obtained, the kinematic analysis can be performed. To do so, Eq.(1) is differentiated with respect to time and the system of constraint equations at velocity level is obtained (2). In (2), $\boldsymbol{\Phi}_q$ is the Jacobian matrix of the constraint vector. The constraint equations at acceleration level can be obtained by differentiating (2) with respect to time (3). In (2) and (3), the sub index t means a partial derivative with respect to time, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ represents the vectors of dependent velocities and accelerations.

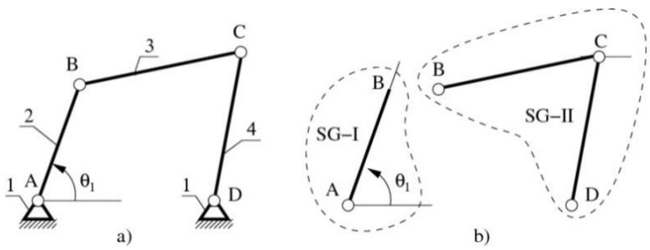
$$\boldsymbol{\Phi}(\boldsymbol{q},t) = 0 \tag{1}$$

$$\boldsymbol{\Phi}_q\dot{\boldsymbol{q}} = -\boldsymbol{\Phi}_t \tag{2}$$

$$\boldsymbol{\Phi}_q\ddot{\boldsymbol{q}} = -\dot{\boldsymbol{\Phi}}_q\dot{\boldsymbol{q}} - \dot{\boldsymbol{\Phi}}_t \tag{3}$$

The number of coordinates increases with the complexity of the multibody system, especially in spatial applications, and the length of the constraints vector $\boldsymbol{\Phi}$ and the size of the Jacobian matrix $\boldsymbol{\Phi}_q$ increases accordingly. To deal with complex problems whose kinematics have to be solved very efficiently any improvement in kinematic formulations must be investigated. The theory of structural analysis gives us the opportunity to explore the efficiency of a topological formulation based on the kinematic structure of the multibody system.

Structural analysis divides a multibody system in a set of kinematic chains, called structural groups (SG), by applying certain topological criteria. Figure 1 shows the division of a four-bar linkage (a) into two structural groups SG-I and SG-II (b). The number of SG, their type and the specific order in which they have been split from the fixed frame is known as the kinematic structure of the multibody system, which can be obtained using graph-analytical or computational methods. In Fig. 1.c, *KinStr* shows the kinematic structure of the four-bar linkage obtained by computational methods [1].



$$KinStr = \begin{pmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Figure 1: a) Four-bar linkage with given input θ_1. b) Division into two structural groups: SG-I and SG-II. c) Computational kinematic structure is represented by *KinStr* matrix which is useful for both kinematic and dynamic analysis [1].

Then, at each time step, the kinematics of the whole multibody system can be determined by solving, in the order defined by its kinematic structure (*KinStr*), the kinematics of each SG. The later can be programmed in separated specific subroutines and solved by selecting a set of *group coordinates* formed by two subsets of dependent ($\mathbf{q}_G$) and independent ($\mathbf{h}_G$) coordinates. Both subsets of coordinates are related to each other by the corresponding *group equations* $\mathbf{\Phi}_G$, whose structure is similar to (1), although strongly reduced [2].

The main objectives of this work are to introduce the topological approach based on SG decomposition for the kinematic analysis of multibody systems, and to compare its efficiency against a global formulation in terms of computing time versus number of constraint equations (*neq*). Two different systems are solved: a scalable four-bar linkage, which allows us to very easily increase the number of constraint equations, and the truck suspension (Figure 2) which introduces different kinds of spatial SG, expanding the applicability of this method to any kind of multibody system. Trucks from two to five-axes are modelled and solved. In order to strictly evaluate the performance of the kinematic formulations, both the topological and the global one use natural coordinates. This introduces a new interesting feature for the topological formulation since most of them use relative coordinates.

Figure 2: Detailed view of an isolated axis and the elements of its suspension system (Upper left). The suspension system mounted on each one of a three-axis truck (Right).

The results obtained show: 1) A similar efficiency of both methods under 100 *neq* and a better efficiency of the method based on structural-group coordinates above this *neq*, 2) these differences might improve for the topological formulation under more powerful programming languages, i.e. Fortran, and 3) this formulation can be extended to 3D multibody systems without any difficulty.

References

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