Interpolating Polynomial Requirements for Path Following Constraints

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Abstract

In the framework of multibody dynamics, the path motion constraint enforces a body to follow a predefined curve being its rotations with respect to a prescribed curve moving frame. For the definition of railway, tramway or roller coaster tracks an accurate description of their geometries is needed. Most often this is done by parameterization of the track centerline where a reference plane at which point the rails sit is required and consequently a curve moving frame must also be specified.

Regardless on the requirement of a general curve geometry being needed to specify either a railway centerline, a rollercoaster spatial geometry or a path motion kinematic constraint, it is not only of importance to select a suitable interpolation scheme but also to use a robust definition of the curve moving frame.

Depending on if the curve is used to set some geometric layout for the mechanical models or for to define kinematic constraints for multibody dynamics applications, higher order derivatives, with respect to the curve parameter, may be required. Therefore, using polynomial interpolation schemes, higher order polynomials may be required for an exact formulation of problem. Generally, higher order interpolating polynomials lead to unwanted, and hardly controllable, oscillation of the curve geometry, i.e., small deviations of the positions of the nodal points lead to large variations of the curve geometry away from those nodes. On the other hand, lower order polynomials generally have a local geometric control but they may not have the order necessary to ensure the proper geometric continuity of the model or the parametric derivatives required in the formulation of a kinematic constraint. Therefore, a question arises where what are the minimal requirements that an interpolating polynomial should meet in order to be used in the definition of a path motion kinematic constraint.

First the geometric description of the curve must allow the definition of a moving frame in which the tangent, normal and binormal vectors define an orthogonal frame. Both Frenet and Dabroux frames are candidates to play the role of the required moving frame [1,2]. As discussed by Tandl and Kecskemethy, both have singularities in general spatial curve geometries, as those required for rollercoaster analysis [3,4]. In this work the Frenet frame is used being the straight segments handled with the provision described by Pombo and Ambrosio [5].

Using the moving frame definition selected for this work a proper formulation for a path motion kinematic constrained is obtained. Such kinematic constraint imposes that a point of a rigid body follows a given curve and that the body itself does not rotate, or does it in a prescribed manner, with respect to the curve moving frame. Depending on the choice of coordinates used on the multibody formulation this kinematic constraint may be defined differently [6,7]. When Cartesian coordinates are used and the equations of motion are solved together with the second time derivative of the position kinematic constraints the definition of the Frenet frame requires the second derivative of the curve with respect to its arc length parameter while the acceleration constraints, i.e., the second time derivative of the kinematic constraint, requires the existence of a fourth derivative. In this sense, apparently fifth order polynomials are required to formulate properly the path motion kinematic constraint.

The numerical integration of the equations of motion of a multibody systems entails the use of numerical integrators, such as Runga-Kutta, Gear or others, to undertake the forward dynamic analysis

[8,9]. All numerical procedures used in the solution of the equations of motion and on their solution have a finite precision and ultimately lead to small errors that affect the precision of the solution. When dependent coordinates, such as Cartesian or Natural coordinates, are used only the acceleration constraints are explicitly used in the solution of the equations of motion being the position and velocity constraints fulfilled only if the numerical integration would be error free, being otherwise violated and leading to instabilities in the dynamic solution of the analysis [6]. By using stabilization procedures, such as the Baumgarte constraint stabilization method [10] or the Augmented Lagrangian Formulation [11] such constraint violations can be kept under control. By using a coordinate partition scheme the constraint violations can be eliminated [12].
The work here presented shows that this same procedures used to stabilize the constraint violations in the integration of the equations of motion of multibody systems formulated with dependent coordinates also allow for the use of interpolation schemes with polynomials that have an order lower than that required for the exact formulation of the path motion kinematic constraints. The presented results show that regardless of using interpolation schemes with higher order polynomials the constraint violations still grow to a point in which either stabilization or coordinate partition procedures are required. Furthermore, regardless of the order of the polynomial actually used for the prescribed motion constraint, when dependent coordinates, such as Cartesian coordinates, are used in the multibody dynamic formulation small numerical errors are always present in the numerical methods used to solve and integrate the equations of motion. This small discrepancies tend to accumulate ultimately leading to the violation of the kinematic constraints of the system. Therefore, the use of constraint stabilization or correction methods is unavoidable independently of the order of the interpolation scheme used. It is shown that when constraint stabilizations methods are used there is no significant difference in the constraint violations between interpolating polynomials of higher and lower order, provided that they satisfy the continuity required for the definition of the curve moving frames and for the geometric requirements of the model.

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