A Comparative study for some semi-recursive methods on real-time multibody vehicle dynamics model

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Abstract

Some topological formulations in terms of semi-recursive fashion have been proposed recently to improve computational efficiency in complex multibody systems (MBS). An improved semi-recursive method has been developed by García de Jalón [1-2], Kim has introduced a subsystem synthesis method [3-4], and Bae has implemented an implicit generalized semi-recursive method [5-8]. These semi-recursive methods, in which the system topology is used to obtain velocities recursively in an open-loop MBS, are efficient for solving the equations of motion of large-scale multibody models.

In this paper, a numerical comparative study among the aforesaid methods is carried out based on theory, accuracy and efficiency. García de Jalón's improved semi-recursive method makes use of a double velocity transformation and Maggi's formulation to derive the equations of motion of the overall system in terms of independent relative coordinates. The first velocity transformation is introduced to transform Cartesian coordinates into relative coordinates. The equations of motion of the open-loop system can be expressed as follows after the first velocity transformation:

$$\mathbf{R}_d^T \mathbf{M}^\Sigma \mathbf{R}_d \ddot{\mathbf{z}} = \mathbf{R}_d^T \left(\mathbf{Q}^\Sigma + \mathbf{P}^\Sigma \right) \tag{1}$$

where, $\mathbf{R}_d$ denotes a diagonal matrix whose elements are velocity vector [1]. $\mathbf{M}^\Sigma$, $\mathbf{Q}^\Sigma$, and $\mathbf{P}^\Sigma$ represent the accumulated inertia matrices, the accumulated external forces and the velocity dependent accumulated inertia forces respectively, and $\ddot{\mathbf{z}}$ is the vector of relative accelerations. The second velocity transformation is applied to replace relative coordinates by independent coordinates, where Maggi's formulation is implemented [2]. The equations of motion in this semi-recursive method have the following form:

$$\mathbf{R}_z^T \mathbf{R}_d^T \mathbf{M}^\Sigma \mathbf{R}_d \mathbf{R}_z \ddot{\mathbf{z}}^i = \mathbf{R}_z^T \mathbf{R}_d^T \mathbf{Q}^\Sigma - \mathbf{R}_z^T \mathbf{R}_d^T \mathbf{M}^\Sigma \left(\dot{\mathbf{R}}_d \dot{\mathbf{z}} + \mathbf{R}_d \dot{\mathbf{R}}_z \dot{\mathbf{z}}^i \right) \tag{2}$$

where, $\mathbf{R}_z$, and $\dot{\mathbf{R}}_z$ are the second velocity transformation matrix computed numerically and its differentiation with respect to time respectively. $\dot{\mathbf{z}}^i$, and $\ddot{\mathbf{z}}^i$ are the vectors of independent relative velocities and accelerations. Furthermore, the rod eliminating method is implemented in this semi-recursive formulation to improve the computational efficiency.

On the other hand, Kim's subsystem synthesis method takes advantage of the special topology structure to decompose large size matrices by means of subsystem partitioning. It also can be illustrated in two parts. The first part consists of formulating the equations of motion of the overall system in terms of Cartesian coordinates of the base body (the chassis) and relative coordinates in the joints in a similar way to equation (1). Then a sub-matrix reduction method is used to construct the equations of motion of the chassis frame and the suspension subsystems separately. The acceleration of chassis frame can be computed as [3]:

$$\left(\hat{\mathbf{M}}_0 + \sum_{i=1}^{4} \breve{\mathbf{M}}_i \right) \dot{\hat{\mathbf{Y}}}_0 = \left(\hat{\mathbf{Q}}_0 + \sum_{i=1}^{4} \breve{\mathbf{P}}_i \right) \tag{3}$$

where, $\hat{\mathbf{M}}_0$, and $\hat{\mathbf{Q}}_0$ are the generalized mass matrix and the force vector of the chassis frame respectively, $\breve{\mathbf{M}}_i$, and $\breve{\mathbf{P}}_i$ denote the effective inertia matrix and the force vector of *i*-th suspension subsystem respectively. $\dot{\hat{\mathbf{Y}}}_0$ represents the vector of acceleration of the chassis frame. The subsystem

accelerations including the Lagrange multipliers can be obtained based on a known vector $\dot{\hat{\mathbf{Y}}}_0$ according to the equations of motion of the subsystem described as follows:

$$\bar{\mathbf{M}}_{ii}\begin{bmatrix}\ddot{\mathbf{q}}\\ \boldsymbol{\lambda}\end{bmatrix}=\bar{\mathbf{P}}_i-\bar{\mathbf{M}}_{0i}\dot{\hat{\mathbf{Y}}}_0 \tag{4}$$

where, $\bar{\mathbf{M}}_{ii}$, $\bar{\mathbf{M}}_{0i}$, and $\bar{\mathbf{P}}_i$ are respectively the generalized mass matrix, the inertia coupling mass matrix and the generalized force vector of the *i*-th suspension subsystem. $\ddot{\mathbf{q}}$, and $\boldsymbol{\lambda}$ are the vectors of relative accelerations and Lagrange multipliers of this subsystem respectively.

Meanwhile, Bae's generalized semi-recursive implicit method involves solving a system of nonlinear equations $\mathbf{H}(\mathbf{p})$, where $\mathbf{p}$ is a vector that includes the relative positions, velocities, accelerations. $\mathbf{H}(\mathbf{p})$ is composed by the dynamic equations; the position, velocity and acceleration constraint equations and the equations of the Backward Differentiation Formula (BDF) integration method. In order to solve system $\mathbf{H}(\mathbf{p})$, the Newton-Raphson approach is employed, thus, the iterative system of linear equations is formulated as follows:

$$\mathbf{H}_{\mathbf{p}_i}\left(\mathbf{p}_{i+1}-\mathbf{p}_i\right)=-\mathbf{H}(\mathbf{p}_i) \tag{5}$$

The key point of Bae's method is that the terms of this system in equations (5) can be computed recursively in an efficient way. It means that, all terms of the form $\mathbf{X}=\mathbf{Bx}$ $\left(\mathbf{X}\in\mathbf{R^n},\ \mathbf{x}\in\mathbf{R^f}\right)$, where n and f denote the number of Cartesian coordinates and degrees of freedom of the mechanism, are evaluated recursively from the root to the leaves. One example of these terms are the Cartesian velocities. On the other hand, all terms of the form $\mathbf{g}=\mathbf{B}^T\mathbf{G}$ $\left(\mathbf{G}\in\mathbf{R^n},\ \mathbf{g}\in\mathbf{R^f}\right)$, such as the forces, are computed recursively from the leaves to the root.

Therefore, in order to compare the numerical efficiency and solution accuracy of these semi-recursive methods, a common implementation framework on the basis of MATLAB has been set. MATLAB is very easy to use for numerical computations, the correctness and accuracy of the results of a full vehicle dynamic simulation using these different formulations may be checked. For the critical parts of the dynamic simulation, a C/C++ MEX file implementation is generated to improve the efficiency of the code significantly. Libraries of sparse, BLAS routines are introduced to accelerate computation with matrices. On the other hand, a similar programming style for the three formulations is used so as to carry out a fair comparison.

References

[1] J. de García Jalón, E. Álvarez, F.A. de Ribera, I. Rodríguez, F.J. Funes: A Fast and Simple Semi-Recursive Formulation for Multi-Rigid-Body Systems. Advances in Computational Multibody Systems, Computational Methods in Applied Sciences 2, 1-23, 2005.

[2] Javier. García de Jalón, Alfonso Callejo, Andrés F. Hidalgo: Efficient Solution of Maggi's Equation. J. Comput. Nonlinear Dynam. 7(2), 021003, 2012.

[3] S. S. Kim: A Subsystem Synthesis Method for Efficient Vehicle Multibody Dynamics. Multibody Syst. Dyn. 7(2), 189-207, 2002.

[4] S. S. Kim, W. Jeong: Subsystem Synthesis Method with Approximate Function Approach for a Real-time Multibody Vehicle Model. Multibody Syst. Dyn. 17(2-3), 141-156, 2007.

[5] Bae D.S., Han J.M., Choi J.H.: An Implementation Method for Constrained Flexible Multibody Dynamics Using a Virtual Body and Joint. Multibody Syst. Dyn. 4(4), 297-315, 2000.

[6] Bae D.S., Lee J.K., Cho H.J., Yae H.: An Explicit Integration Method for Realtime Simulation of Multibody Vehicle Models. Methods Appl. Mech. Engrg. 187(1-2), 337-350, 2000.

[7] Bae D.S., Cho H., Lee S., Moon W.: Recursive Formulas for Design Sensitivity Analysis of Mechanical Systems. Methods Appl. Mech. Engrg. 190(29-30), 3865-3879, 2001.

[8] Bae D.S., Han J.M., Choi J.H., Yang S.M.: A Generalized Recursive Formulation for Constrained Multibody Dynamics. Int. J. Numer. Methods Eng. 50(8), 1841-1859, 2001.