On Attitude Dynamics and Control of Legged Robots Using Tail-Like Systems

Konstantinos Machairas and Evangelos Papadopoulos

School of Mechanical Engineering
National Technical University of Athens
9 Heroon Polytechniou Str., 15780 Athens, Greece
[kmach, egpapado]@central.ntua.gr

Abstract

Over the last five years, research in legged robotics has drawn again the attention of the robotics research community, as universities and companies came up with impressive accomplishments, especially in the field of quadrupedal locomotion. Although much research has been conducted concerning the design and control of legs of various morphologies, attitude control of the body is yet poorly investigated. However, most of the tasks assigned to legged robots, such as high speed galloping or jumping over obstacles, require precise control of the robot's attitude. So far, attitude control is mostly achieved indirectly through the motion of the legs, a technique that assigns more control tasks to the legs forcing them to trade-offs that may lead to low performance.

To better mitigate this challenge, dedicated appendages with greater moment of inertia (MoI) can be used. Once again, ideas can be derived from biology; one quickly thinks of animal tails. Many quadruped mammals have long tails, which aid to balance and maneuver at high speeds. Kangaroo rats use their long tails for righting and turning in midair. Black rats can impressively enter a building by balancing along a 2 mm wire. Moreover, studying hopping by kangaroos, one may be amazed to see how they use their tails to counteract the body pitching induced by the motion of their legs. In general, legged animals mostly use their tails for fine adjustments to perturbations, when their legs are otherwise occupied, [1].

While numerous legged robots have been designed, only a minority employ dedicated appendages for angular momentum management, such as tails or reaction wheels, [2-4]. To the authors' knowledge, no methodology has been proposed concerning the design of such tail-like mechanisms. Here, we introduce a design methodology based on analytical expressions derived from the system dynamics, while we clarify several issues concerning the holonomy of the system and its implications on attitude control.

To begin with, we introduce a simple planar template of two coupled bodies i.e. a body and a tail (Figure 1). By body we mean the body with the four legs and the head of the robot, except for the tail. This is a reasonable assumption if one considers zero leg MoI and a rigid spine. Since a reduction to the system center of mass (CoM) frame is possible, we choose to parameterize the configuration space only by the absolute pitch angle of the body $\theta \in S^1$, and the relative hinge (motor) angle of the tail $q \in S^1$. Let (m_0, I_0) and (m_1, I_1) denote the mass and the MoI about its CoM, for the body and the tail respectively. Let r be the distance from the body's CoM to the joint, and l be the distance from the tail CoM to the joint. Finally, let τ be the control torque that the body exerts on the tail, with the motor modeled as an ideal torque source. The equations of motion (EoM) of the system are:

$$\begin{aligned}&\left(I_0+I_1+\mu\left(l^2+r^2+2rl\cos q\right)\right)\ddot{\theta}+\left(I_1+\mu\left(l^2+rl\cos q\right)\right)\ddot{q}-\mu rl\sin q\dot{q}^2-2\mu rl\sin q\dot{q}\dot{\theta}=0\\&\left(I_1+\mu l^2+\mu rl\cos q\right)\ddot{\theta}+\left(I_1+\mu l^2\right)\ddot{q}+\mu rl\sin q\dot{\theta}^2=\tau\end{aligned} \qquad (1)$$

The two masses appear only in the form of an important quantity $\mu=(m_1m_2)/(\mathrm{m}_1+\mathrm{m}_2)$, that we call the system's *effective mass.* We note here that the generalized momentum associated with the cyclic coordinate θ is conserved yielding:

$$\left(I_0+I_1+\mu\left(l^2+r^2+2rl\cos q\right)\right)\dot{\theta}+\left(I_1+\mu\left(l^2+rl\cos q\right)\right)\dot{q}=h_0 \qquad (2)$$

which is in fact the equation for the conservation of the system's angular momentum about its COM, with h_0 being the system initial angular momentum. Equation (2) can take the form of an acatastatic Pfaffian constraint, which is nonholonomic only when $r, l \neq 0$ and $h_0 \neq 0$. Practically, this means that for zero initial angular momentum, the conservation equation is analytically integrable independent of the position of the joint. When time enters as a third variable through the initial angular momentum, the constraint can be integrated only if the tail is pinned at the body's CoM ($r=0$). A holonomic constraint is in fact a geometric one and thus each θ corresponds to a specific q; a nonholonomic constraint makes the whole configuration manifold accessible, and any pair (θ, q) can be achieved.

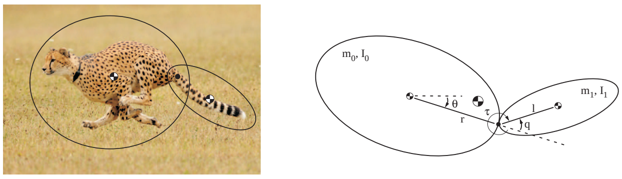

Figure 1: A Cheetah during flight phase and a two-body template.

The above analysis is important for the attitude control of a legged robot, and provides the basic guidelines for building a design methodology for tail-like systems. On this basis, we propose steps for selection of the basic parameters of such systems. The main parameters to be selected are the mass, the MoI and the length of the tail-like appendage, while the motor characteristics are also of great importance. Depending on the case, several criteria can be used for the calculation of these parameters. These concern: (a) the maximum change of the body angle that can be achieved during a flight phase, (b) the maximum body angular velocity that can be rejected through the tail's motion, (c) the maximum accelerating or decelerating force appearing at the tail joint, and (d) the maximum change on body's angular momentum induced by leg motion.

Here, preliminary simulation results are presented for $m_0=30\ kg$, and $I_0=2\ kgm^2$, which are realistic for a cheetah and a robot as well. To compute the appropriate characteristics of the tail, and considering no initial angular momentum, we assume that a reasonable maximum change in pitch angle would be $\Delta\theta=15^o$, with a maximum tail rotation of $\Delta q=180^o$, and a time interval almost equal to the swing time of a Cheetah's leg i.e. $t=0.2\ sec$. With these parameters, a suitable tail mass can be computed ($m_1=1\ kg$) based on the previous dynamic analysis, and after considering the tail's length being almost equal to the length of a robot's leg ($l=0.4\ m$). Some preliminary results appear in Figure 2, where we start the body with initial angular velocity of *0.6 rad/sec*. With the tail fixed the body's final pitch angle at the end of the flight phase is unacceptably big. When the tail is used it absorbs the kinetic energy from the body, assuring that the robot lands with an appropriate angle.

Regarding the final paper, the dynamic analysis and the design methodology will be presented in detail, while numerical simulations will be carried out with more complex dynamic models, such as quadruped robots with multi-link legs and tail-like appendages. Finally, useful conclusions will be drawn about whether a tail-like system can justify the extra weight added to the robotic system.

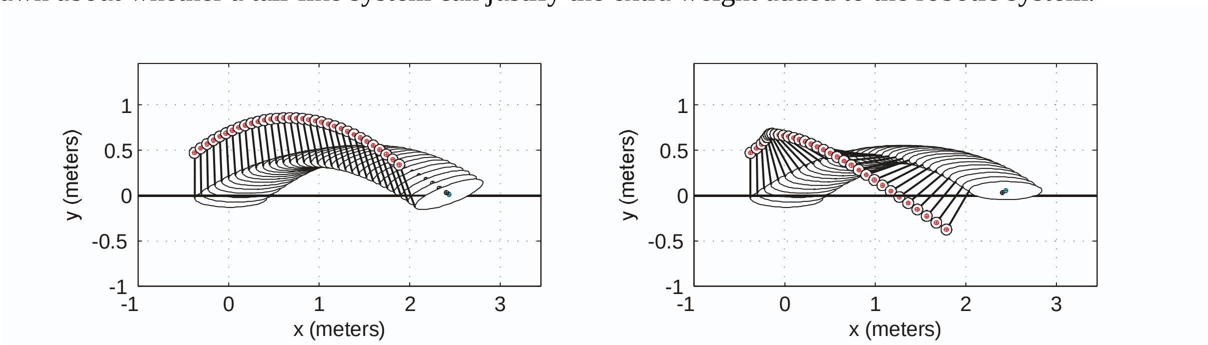

Figure 2: **Left:** The robot in flight phase with initial body angular velocity 0.6 rad/sec and fixed tail, **Right**: The robot lands with zero body angular velocity due to the controlled motion of the tail.

References

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