A geometrically exact shell finite element with an almost constant tangent stiffness matrix

Valentin Sonneville*, **Alberto Cardona**# and **Olivier Brüls***

*Department of Aerospace and Mechanical Engineering (LTAS)
University of Liège
Chemin des Chevreuils 1 (B52/3), 4000 Liège, Belgium
v.sonneville,o.bruls@ulg.ac.be

#CIMEC
Universidad Nacional Litoral - Conicet
Colectora Ruta Nac 168 / Paraje El Pozo, 3000 Santa Fe, Argentina
acardona@intec.unl.edu.ar

Abstract

In this work, a geometrically exact shell finite element is formulated using an innovative differential-geometric framework. The same framework has been exploited in past publications for the modelling of rigid bodies[1], kinematic joints[2], and beams[3]. Under the shell assumption, the general study of a three dimensional continuum can be reduced to a two-dimensional study on the so-called neutral surface of the shell and a-one dimensional study over the thickness. The configuration of a point of the neutral surface of the shell is described by a mapping $\mathbb{R} \times \mathbb{R} \to SE(3) : (\alpha_1, \alpha_2) \to \mathbf{H}(\alpha_1, \alpha_2)$

$$\mathbf{H}(\alpha_1, \alpha_2) = \begin{bmatrix} \mathbf{R}(\alpha_1, \alpha_2) & \mathbf{x}(\alpha_1, \alpha_2) \\ \mathbf{0}_{3\times 1} & 1 \end{bmatrix} \tag{1}$$

where $\mathbf{x}(\alpha_1, \alpha_2)$ is the absolute coordinate of the point and $\mathbf{R}(\alpha_1, \alpha_2)$ is a rotation matrix which describes a local orientation with respect to the inertial axes. Here, $SE(3)$ denotes the special Euclidean group. The derivative with respect to time t can be expressed in the form

$$\frac{\partial \mathbf{H}}{\partial t} = \mathbf{H}\tilde{\mathbf{v}} \tag{2}$$

where $\tilde{\mathbf{v}}$ is a velocity variable which is naturally expressed in the local frame and which can be interpreted as a left-invariant vector field in the Lie group theory. Similarly, the deformation gradient can be introduced by replacing the derivative with respect to time by a derivative with respect to a spatial coordinate. As a consequence, the deformation gradient is naturally expressed in the local frame. Eventually, the equations of motion can be expressed in terms of the local velocities and local deformations only and are thus invariant under rigid body motions.

Since full three-dimensional rotation matrices are used, the drilling phenomenon is present. Thanks to a rigorous discussion of the rank of the 12×12 stiffness matrix and of the geometric compatibility conditions of the 12 deformation measures used in the formulation, the drilling phenomenon is clearly identified at the continuous level. In particular, it is shown that there exists a zero-energy deformation mode when the two local curvatures of the reference surface are equal, i.e., the shell is locally a plate or a sphere.

Consistent spatial interpolation formulae are then introduced for the finite element discretization. By construction, this interpolation respects the nonlinear and non-commutative nature of the group $SE(3)$. Also, they only rely on relative transformations between the nodal frames so that they are naturally invariant under rigid-body motion.

Finally, the semi-discrete equations of motion are formulated as a differential equation on a Lie group, which is solved using a Lie group implicit time integrator [4, 5]. It turns out that the unknowns and residuals that are involved at the level of the Newton iterations are all expressed in the local material frame. As a consequence, the tangent stiffness matrix remains strictly constant when the element undergoes rigid body motions and almost constant when the deformations of the element are small.

The proposed formulation is applied to standard shell benchmarks in order to exhibit its consistency, its accuracy and the absence of locking problems.

Acknowledgement

Valentin Sonneville would like to acknowledge the Belgian National Fund for Scientific Research (FRIA) for its financial support.

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