Parallel recursive Hamiltonian formulation for constrained multibody system dynamics

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Abstract

This paper presents a novel recursive formulation for the simulation of constrained multibody system dynamics based on the Hamilton's canonical equations. The systems under consideration are subjected to holonomic constraints in generalized topologies, i.e.: serial chains, tree chains or closed-loop topologies. The Hamilton's canonical equations exhibit many advantageous features compared to their acceleration based counterparts [1]. From the literature review [2], it also appears that there is a lack of parallel algorithms for general multi-rigid body system dynamics based on the Hamiltonian approach. We consider the Hamilton's equations in the following standard form:

$$\dot{\mathbf{q}} = \left(\frac{\partial \mathscr{H}}{\partial \mathbf{p}}\right)^T \tag{1a}$$

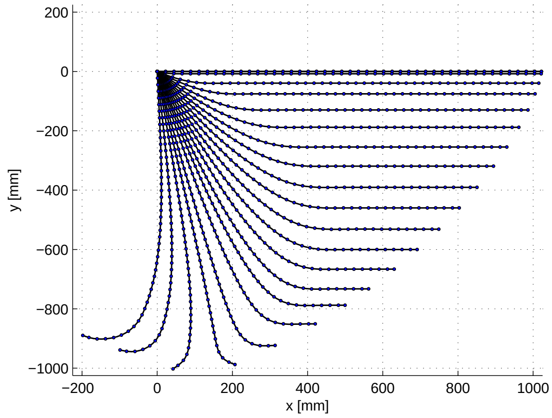
$$\dot{\mathbf{p}} = -\left(\frac{\partial \mathscr{H}}{\partial \mathbf{q}}\right)^T + \mathbf{Q} - \mathbf{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} \tag{1b}$$

$$\mathbf{\Phi}(\mathbf{q}) = \mathbf{0} \quad \Rightarrow \quad \dot{\mathbf{\Phi}}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} = \mathbf{0} \tag{1c}$$

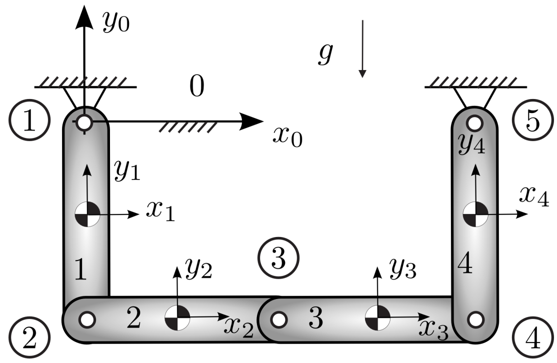
where $\mathscr{H}$ – is the Hamiltonian, $\mathbf{q}$, $\mathbf{p}$ – canonical generalized coordinates, $\mathbf{Q}$ – external forces, $\mathbf{\Phi}_{\mathbf{q}}^T$ – Jacobian matrix that shows the constrained directions, $\boldsymbol{\lambda}$ – Lagrange multipliers. The standard set of Hamilton's canonical equations (1) is slightly reformulated throughout the algorithm development to conveniently make use of mixed canonical absolute and relative coordinates for the system state description. Specifically, Eq. (1a) is supplemented by the impulsive constraint forces $\boldsymbol{\sigma}$ (where $\dot{\boldsymbol{\sigma}} = \boldsymbol{\lambda}$) and joined together with the constraint equations at the velocity level (1c). The resulting algebraic equations may be set together in the form of linear equations in terms of joint velocities and impulsive constraint forces at joints. On the other hand, the equations of motion (1b) are somewhat rearranged in order to obtain a direct dependence on the impulsive loads at joints.

The developed formulation leads to a two-stage procedure. In the first phase, the approach utilizes divide and conquer algorithm (DCA) , i.e. a hierarchic assembly-disassembly process by traversing the multibody system topology in a binary tree manner [3], [4]. The objective is to solve the equations for the velocities and constraint impulsive loads at joints given the positions and momenta at joints. The process exhibits linear $O(n)$ (n – number of bodies) and logarithmic numerical cost, in serial and parallel implementations, respectively. The time derivatives of the total momenta are directly evaluated in the second step of the algorithm at the $O(n)$ expense, sequentially, and at the constant cost $O(1)$, in parallel. The algorithm is exact and non-iterative possessing many favorable features over the acceleration based counterparts [3], [5].

Sample open- (Fig. 1(a)) and closed-loop (1(b)) test cases indicate excellent constraint satisfaction at the position and velocity level as well as marginal energy drift without any additional form of the constraint stabilization techniques involved in the solution process. These results are comparatively set against the solution collected from the more standard acceleration based formulations, MSC.ADAMS commercial software, and the results taken from real-life physical experiment [6] for open-loop chain. Some sample numerical results for the closed-loop pentagonal system are presented in Fig. 2(a), and 2(b)

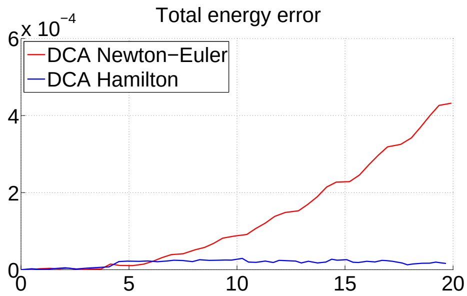


(a) Open-loop multibody system

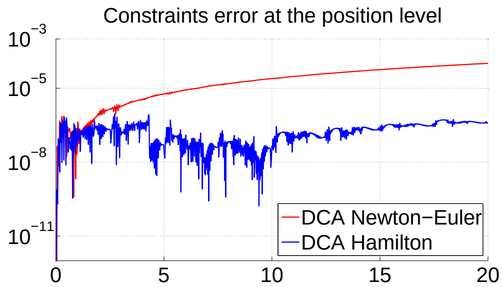


(b) Closed-loop multibody system

Figure 1: Sample multibody test cases: (a) open-loop system possessing 229 bodies falling under gravity forces, (b) a representative closed-loop system



(a) Energy



(b) Constraints

Figure 2: Performance of the algorithm: (a) Energy drift for the closed-loop system, (b) Constraints violation errors for the closed-loop chain

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Acknowledgments

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