

# Multi-Body Dynamics Benchmarks for Frictionless Elastic Collisions

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## Abstract

Multi-body simulations are used for predicting performance of a system, or for providing training in a realistic environment. Most of the research on these simulators investigate accuracy, efficiency and robustness. Benchmarks are defined to check these properties in order to quantify to what extent they are fulfilled. Examples of benchmarks for constrained systems and friction exist, but they are lacking for contact problems. Five new possible benchmarks are given, which can be used to test the accuracy of elastic frictionless collision response. Conservation of momentum is used to derive the analytical solution, which is shown to yield the same results as standard textbook examples. Results of a multi-body simulation are compared with the analytical results.

## 1 Introduction

Multi-body simulations are often employed to engineer a system for a certain scenario, or to train people to operate equipment. For many-real life applications the simulations are performed using multi-body dynamics. It seems like most research in this area is devoted to efficiency and robustness of the algorithms, as previously stated by González et al[1, 2]. A number of quantitative benchmark cases exist for multi-body problems, which are collected in a database by Masoudi et al[4]. These cases are all dynamic, conservative and test the dynamics engine on its holonomic constraint handling. This means these benchmark cases do not consider contact problems. Some benchmark cases which evaluate contact behaviour, such as by Seugling and Rölin[3], are purely qualitative.

In this paper benchmarks are proposed for frictionless elastic collisions. First, a description is given of the new benchmarks. Then, for comparison and verification a suitable analytical solution is derived. The goal is to use the results of the simulations for training and engineering of maritime operations in ice and heavy-lifting.

## 2 Proposed benchmark

Five benchmarks are proposed, each consisting of two building blocks. One big block, depicted in Figure 1 as blue and a number of small blocks (in red). By taking different impact locations of the small blocks onto the big block the response of the blocks is different. The small blocks are all of equal mass and have an initial velocity perpendicular to the surface they are going to hit making sure no friction takes place; the big block is at rest in its starting position. The mass of the big block can be chosen to be most relevant, for instance for ice simulations the big block is much heavier than the smaller block, since the ice floes are typically lighter than the ship. No external forces act on any of the bodies, so there is only one physical process: collision. Table 1 shows the qualitative response of the five different configurations. The coordinates are Cartesian, and the angles  $\alpha$ ,  $\beta$  and  $\gamma$  are rotations  $y-z$ ,  $z-x$  and  $x-y$  axes respectively.

## 3 Analytical expression for response

The derivation of the exact response is a slightly reformulated version of Ermolin and Kazakov [5]. The formulation is based on conservation of linear and angular momentum. All collisions are assumed to take place at the same time. For each collision two bodies are identified, which both experience the same

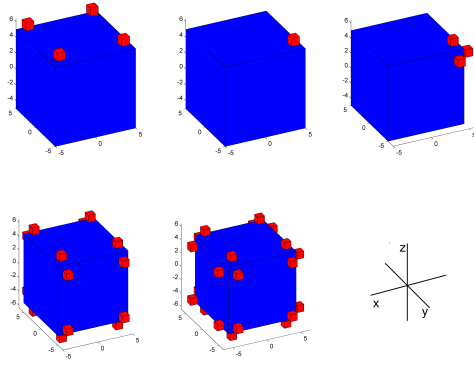


Figure 1: Five proposed benchmarks for validation of rigid body software and the coordinate system.

impulse. For each collision a constraint is set, which prevents the bodies from interpenetration. For a system of collisions a collision matrix  $\mathbf{A}$  can be defined:

$$\mathbf{A}\mathbf{p} = \mathbf{b} \quad (1)$$

with  $\mathbf{p}$  as impulse vector having one component for each collision and  $\mathbf{b}$  as initial configuration. The components of  $\mathbf{A}$  denote the effect collisions have on another through bodies, as if it were masses connected by springs. The impulse  $\mathbf{p}$  is now calculated and applied to each body, which results in a new terminal velocity. By comparing this with the terminal velocity of the multi-body simulation the accuracy is calculated.

## References

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Table 1: Expected terminal velocity of the big block after collision with the small blocks. A + denotes a positive velocity, - a negative velocity and 0 no velocity.

Case	Configuration	$\dot{x}$	$\dot{y}$	$\dot{z}$	$\dot{\alpha}$	$\dot{\beta}$	$\dot{\gamma}$
1	Four blocks on one side	0	0	-	0	0	0
2	One block on one edge	0	0	-	+	+	0
3	Three blocks on one edge	-	+	-	0	0	0
4	Sixteen blocks symmetric on four sides	0	0	0	0	0	0
5	Twenty-four blocks symmetric on six sides	0	0	0	0	0	0