## The Motion Formalism in Multibody Dynamics

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## Abstract

Simulation software for flexible multibody systems (FMS) is becoming an increasingly important design tool for engineering systems. Many commercial tools for FMS simulation are based on the floating frame of reference formulation, which decomposes the motion of the structure into rigid-body and small elastic motions. Because increasingly complex configurations are investigated, the finite element method (FEM) has become an integral part of the simulation process. The state-of-the-art approach is to use FEM tools to evaluate the eigenmodes of the system, which are used to represent the small elastic motions in the FMS analysis. Once the dynamic loads are predicted by the FMS tool, they are applied in a quasi-static manner back onto the FE model to evaluate 3D stress distributions in the composite layers. This approach is time consuming and its accuracy cannot the assessed. Furthermore, it does not integrate FEM and FMS algorithms, a process that has remained elusive because no satisfactory formulation has emerged that can deal with both FEM and FMS efficiently. Consequently, the efforts of leading companies to develop accurate, reliable, and efficient tools for FMS dynamics analysis have failed, leaving unmet an important need of industry.

In the analysis of rigid multibody systems (RMS), the core difficulty lies in the representation of the kinematics of the system rather than the simulation of its dynamic behavior. The traditional approach has been to decompose motion into displacement and rotation, using the rotation tensor to represent the latter. Due to the complexity of manipulating rotations, which do not form a linear space, the last three decades have witnessed the development of several "rotationless formulations." Among these, the "natural coordinate" approach developed by García de Jalón *et al.* [1, 2] for planar mechanisms, which was later expanded to spatial mechanisms [3, 4], stands out.

When dealing with FMS, the analysis becomes far more complex. Cosserat solids, such as beams, plates, and shells are the basic structural components and the description of their kinematics calls for both displacement and rotation fields in 1D or 2D. Naturally, the FEM is used to deal with these problems and the approach of choice is the co-rotational formulation [5], which is an adaptation of the FFR method to the FEM framework. One node of the element defines the local material frame, *i.e.*, represents the element rigid-body motion, and the FE discretization describes the local deformation in this material frame. This approach, however, inherits the deficiencies of the FFR formulation, and furthermore, is unable to represent rigid-body motions exactly.

Screw theory, first introduced by Ball [6] in 1900, is a fundamental tool of kinematics and robotics. Its central finding is that the displacement and rotation fields of a rigid body cannot be described independently. Yet all textbooks in multibody systems dynamics represent the motion of rigid bodies using independent displacement and rotation fields. The same contradiction is found in the formulation of Cosserat solids (beams, plates, and shells) and mechanical joints that form FMS. The basic lesson of screw theory is ignored by the multibody dynamics community.

The proposed *motion formalism*, so named because it is based on the motion tensor, generalizes screw theory to FMS. Because the motion tensor is suitable to describe the kinematics of both FMS and FEM, the proposed approach bridges the gap between the two formulations naturally. This assertion hides a fundamental theoretical hurdle. Motions do not form a linear space but rather a Lie group, and hence the linear interpolation process inherent to the FEM does not respect intrinsic Lie group properties. Even for the simpler problem of rotations, numerous publications have documented the problems associated with interpolation and the lack of objectivity of the resulting strain measures, see Crisfield and Jelenić [7]. The same concerns arise with the interpolation of motion. Bauchau and Han [8] have investigated the problem and concluded that although the interpolation process must be performed carefully and must

satisfy specific criteria, it is physically meaningful and is as accurate as the interpolation of displacement, a basic tool in FEM.

While rooted in a new kinematic description of FMS, the proposed motion formalism results in a novel Eulerian formulation of dynamics. Theoretical advantages follow: the equations of motion are cast in a flux preserving form and the preservation of invariants such as energy and momentum is underlined. Numerical advantages include robust time integration schemes, exact preservation of invariants, and new shape functions that improve the finite element performance dramatically.

The problem of interpolating motions also impacts time integration schemes for RMS. The consequence of ignoring the intrinsic nature of motions is significant whenever the motion field, *i.e.*, the rotation and displacement, is interpolated, for instance within time integration of RMS models in absolute coordinates. This problem is traditionally disregarded since, although the incorrect motion interpolation increases the absolute numerical integration error, but it does not impair the order of accuracy of the integration method. However, the error becomes significant for the satisfaction of geometric constraints, which can be eliminated by a correct representation of motions [9]. Insofar the proposed motion formulation serves for FMS as well as for RMS.

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