

# Uncertainty Propagation Analysis of Flexible Multibody Systems considering Random Fields

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## Abstract

In the most of practical engineering problems, properties of system parameters in a flexible multibody system such as density, geometries of components, stiffness, damping coefficient and etc. have uncertainties because of manufacturing tolerances, material irregularities and any other uncertain factors. Since the uncertainties of the system parameters cause the uncertainty of a system performance, the uncertainties of the system parameter should be considered for reliable system design. To conduct reliability design of a flexible multibody system, statistical methods can be employed. For the case of the flexible multibody system, the uncertainty of the system parameters are random fields. Compared to random variables, random fields have random function with respect to spatial variables. To conduct uncertainty propagation considering the random fields of the system parameters, a polynomial chaos expansion can be used. The polynomial chaos expansion is given as follows: [1]

$$u(\mathbf{x}, \boldsymbol{\theta}) = \sum_{k=0}^{\infty} u_k(\mathbf{x}) \psi_k(\boldsymbol{\xi}(\boldsymbol{\theta})) \approx \sum_{k=0}^S u_k(\mathbf{x}, t) \psi_k(\boldsymbol{\xi}(\boldsymbol{\theta})) \quad (1)$$

$$S + 1 = \frac{(N + p)!}{N! p!} \quad (2)$$

where,  $u_k(\mathbf{x})$  are deterministic functions,  $\psi_k(\boldsymbol{\xi})$  are polynomial basis functions,  $\boldsymbol{\xi}(\boldsymbol{\theta})$  is a random variable vector,  $N$  is the number of random variables and  $p$  is an order of the polynomial basis function. The random fields of the system parameters can be represented by Eq. (1). Using the random fields of the system parameters, the random fields of the response of the flexible multibody systems can be obtained using Galerkin projection. [2, 3]

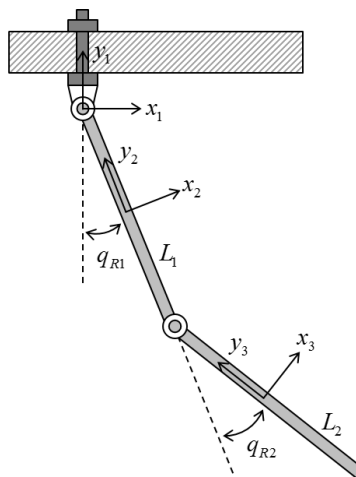


Figure 1: Double pendulum having flexible links.

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Figure 1 shows a double pendulum having flexible links. Each link has a rectangular cross section and a width and a height of the cross section of each link are assumed as random fields as follows:

$$b(x, \theta) = \sum_{l_b=0}^{M_b} b_{l_b}(x) \xi_{l_b}(\theta) = \bar{b} + 0.05\bar{b} \xi_1 \sin\left(\frac{10\pi x}{L}\right) \quad (3)$$

$$h(x, \theta) = \sum_{l_h=0}^{M_h} h_{l_h}(x) \xi_{l_h}(\theta) = \bar{h} + 0.05\bar{h} \xi_1 \sin\left(\frac{10\pi x}{L}\right) \quad (4)$$

Using the Galerkin projection, random fields of the transient responses and eigenvalues of the double pendulum are obtained. The accuracy of the obtained random fields of the transient responses and eigenvalues were verified by comparing MCS results.

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### References

- [1] Ghanem R. G., Spanos P. D., Stochastic Finite Elements: A Spectral Approach, Springer, 1991.
- [2] Jardak M., Su C. H., Karniadakis G. E., Spectral Polynomial Chaos Solutions of the Stochastic Advection Equation, Journal of Scientific Computing, Vol. 17, No. 1-4, pp. 319-338, 2002.
- [3] Kewlani G., Crawford J., Iagnemma K., A polynomial chaos approach to the analysis of vehicle dynamics under uncertainty, Vehicle System Dynamics, Vol. 50, No. 5, pp. 749-774, 2012.