Three-Dimensional Nonlinear Shell Theory for Flexible Multibody Dynamics

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Abstract

In flexible multibody systems, many components are approximated as shells. More often that not, classical shell theories, such as Kirchhoff or Reissner-Mindlin shell theory, form the basis of the analytical development for shell dynamics. The advantage of this approach is that it leads to a simple kinematic representation of the problem: the shell's normal material line is assumed to remain straight and its displacement field is fully defined by three displacement and two rotation components. While such approach is capable of capturing the kinetic energy of the system accurately, it cannot represent the strain energy adequately. For instance, it is well known from three-dimensional elasticity theory that the normal material line will warp under load for laminated composite shell, leading to three-dimensional deformations that generate complex stress states. To overcome this problem, several high-order, refined plate and shell theories [1, 2, 3] have been proposed. While these approaches work well for some cases, they typically lead to inefficient formulation because they introduce numerous additional variables.

In this paper, a semi-discretization of the general equations of three-dimensional nonlinear elasticity is performed, defining the "local model." The equations relating the stress resultants, the sectional deformations, and the warping field for the normal material line are derived from a linear combination of the equations of the local model. These equations define the relationship between the stress resultants the sectional deformations in an implicit manner, and hence, define the "global model." A set of power series solutions are found for the combined equations of local and global models. They are obtained through a recursive solution of the combined equations.

Based on these solutions, the three-dimensional nonlinear elasticity equations of shell-like structures can be reduced to the nonlinear, two-dimensional, geometrically exact shell equations, and the three-dimensional stress field can recovered from the two-dimensional shell solutions. In the reduction process, a 9×9 sectional stiffness matrix is determined, which takes into account the warping effects due to material heterogeneity. In the recovery process, three-dimensional stress field at any point in the shell can be recovered from the two-dimensional shell solution. The proposed method is valid for anisotropic shells with arbitrarily complex, through-the-thickness lay-up configurations.

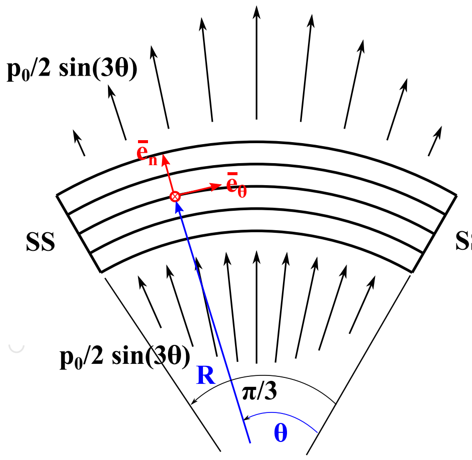

Figure 1: A cylindrical shell under pressure.

To illustrate the proposed approach, a composite cylindrical shell under bending is investigated. Figure 1 shows the configuration of the problem. The shell is simply supported along its two edges at $\theta = 0$ and $\pi/3$, and is subjected to the normal traction $q(\theta) = p_0/2\sin(3\theta)$ along the normal $\bar{e}_n$ direction, over both lower and upper surfaces. The outer, mean, and inner radii of the shell are denoted R_o, R_m, and R_i, respectively. The shell is of thickness $h = R_m\phi/4$, but of infinite length along unit vector $\bar{e}_z = \bar{e}_n \times \bar{e}_\theta$. The through-the-thickness lay-up consists of 4 plies of identical material, each of thickness, $t_p = h/4$. The material has the following stiffness properties: the longitudinal, transverse, and shear moduli are

$E_L = 25$, $E_T = 1$, $G_{LT} = 0.5$, and $G_{TT} = 0.2$ Msi, respectively; the Poisson's ratios are $\nu_{LT} = 0.25$ and $\nu_{TN} = 0.25$. For this problem, analytical solutions were obtained by Pagano [4]. The following lay-up is considered: $[30°, -30°, -30°, 30°]$. The lay-ups defined in fig. 1 start with the bottom ply and end with the top ply; 0° fibers are aligned with unit vector $\bar{e}_z$ and a positive ply angle indicates a right-hand fiber rotation about unit vector $\bar{e}_n$.

In this example, a single, four-noded one-dimensional element was used to model each ply. In the stress recovery process, quadratic expansion of external loads and stress resultants are investigated. Exact stress resultants and their derivatives are obtained by integrating the three-dimensional stress along the shell's thickness. Figures 2, 3, 4, and 5 show the distribution of non-dimensional hoop, axial, normal, and shear stress components at the mid-span of the shell, respectively. The proposed solutions of stress components are in excellent agreement with the analytical solutions.

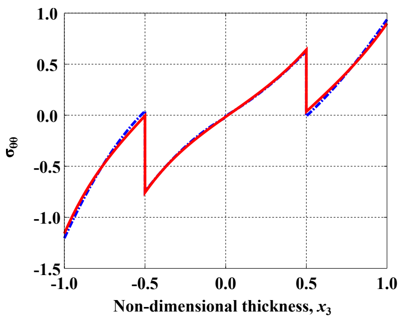

Figure 2: Distribution of non-dimensional hoop stress component, $\sigma_{\theta\theta}/p_0$, through the shell's thickness. Exact solution: dashed-dotted line; present solution: solid line.

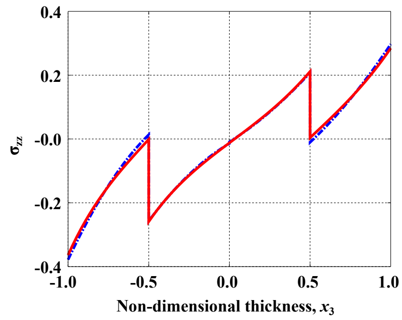

Figure 3: Distribution of non-dimensional axial stress component, σ_{zz}/p_0, through the shell's thickness. Exact solution: dashed-dotted line; present solution: solid line.

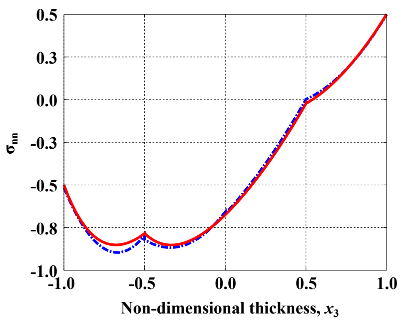

Figure 4: Distribution of non-dimensional normal stress component, σ_{nn}/p_0, through the shell's thickness). Exact solution: dashed-dotted line; present solution: solid line.

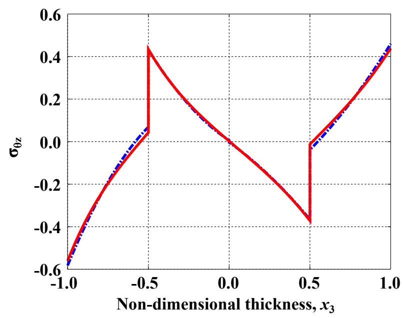

Figure 5: Distribution of non-dimensional shear stress component, $\sigma_{n\theta}/p_0$, through the shell's thickness. Exact solution: dashed-dotted line; present solution: solid line.

References

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