Numerical integration of underactuated mechanical systems subjected to mixed holonomic and servo constraints

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Abstract

We present a new approach to the inverse dynamics simulation of discrete mechanical systems. The present approach is relying on the use of servo constraints for the partial specification of the motion of mechanical systems (see, for example, [1, 2, 3]). In particular, we focus on the specification of trajectories of specific points of a multibody system such as the end effector of a robot.

The partial specification of the motion of a multibody system by means of servo constraints typically leads to a problem formulation in terms of differential-algebraic equations (DAEs). If minimal coordinates are used, the differential part of the DAEs corresponds to the equations of motion while the algebraic part is related to the servo constraints. The servo constraints enforce the desired motion along prescribed trajectories and thus specify the control outputs of the system. To determine the associated control inputs required to steer the system such that the prescribed trajectories are tracked, the DAEs need to be solved. In this way, a simulation approach to the feedforward control of multibody systems can be realized.

In the special case of fully actuated multibody systems the simulation approach to the inverse dynamics problem yields index-3 DAEs that can be integrated in analogy to the DAEs corresponding to constrained mechanical systems (see, for example, [4]). However, the situation changes considerably if underactuated mechanical systems are dealt with. In this type of systems the number of degrees of freedom exceeds the number of controls. Examples of underactuated systems are cranes and flexible multibody systems. The use of servo constraints in the context of underactuated multibody systems leads to a broad diversity of servo constraint problems (see, in particular, the recent papers [5, 6, 7]). One indicator of problem diversity is the (differentiation) index of the underlying DAEs that typically ranges from three to five and even higher. Consequently, to facilitate a stable numerical integration some kind of index reduction approach needs to be applied.

In the present work we newly propose to apply a specific index reduction technique called minimal extension (see [8]). Our approach can be viewed as an alternative to the projection method developed in [9]. Index reduction by minimal extension is based on the introduction of new algebraic variables along with the enlargement of the DAEs by appending time derivatives of the constraints.

We show that index reduction by minimal extension can be applied very efficiently by exploiting the specific structure provided by the mechanical systems under consideration. In this connection either minimal coordinates or redundant coordinates can be used. One sample application deals with a family of crane models that are known to belong to the class of differentially flat systems. In a first step the minimal extension approach can be used to lower the index of the DAEs from five to three. In a second step the index can even be reduced to one.

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