Computation of shortest musculotendon paths using natural geodesic variations

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Abstract

In musculoskeletal simulation, musculotendon paths are commonly modeled as locally length-minimizing strings between muscle origin and insertion. An accurate computation of these paths, their length and their rate of length change is essential to the prediction of muscle moment arms, muscle forces, and the resulting joint loads. A single muscle typically wraps around multiple complex obstacles, yet state-of-the-art muscle wrapping methods are either limited to analytical results for a pair of simple surfaces [1], or they are computationally expensive as, e.g., they use optimization algorithms [2]. Here we present a method that allows for both the fast and accurate computation of a musculotendon's shortest path across an arbitrary number of general smooth wrapping surfaces. The method uses a root-finding algorithm with an explicit Jacobian and computes high-precision solutions for path length and rate of length change, allows for wrapping over biologically accurate surfaces, and is capable of simulating muscle paths over hundreds of surfaces in real-time.

Consider the shortest path between two points that wraps around an ordered set of n obstacle surfaces S^i. Since the path length is minimal, the total path can be regarded as a concatenation of $n+1$ straight-line segments that connect collinearly to n local surface geodesics γ^i. The collinearity conditions at the transitions between geodesics and adjacent straight straight lines are used to formulate a nonlinear constraint equation for the path error $\varepsilon \in \mathbb{R}^{4n}$, whose root defines the shortest path [3]

$$\varepsilon(q) = 0\,, \quad q = \begin{bmatrix} q^1 & \dots & q^i & \dots & q^n \end{bmatrix}, \tag{1}$$

where q are the parameters defining the geodesics. Here, each geodesic is naturally parameterized by its starting point, direction, and length. This parameterization yields $q \in \mathbb{R}^{5n}$ on parametric surfaces and $q \in \mathbb{R}^{7n}$ on implicit surfaces. In both cases, Eqn. 1 has more unknowns than equations. Hence, a direct gradient-based solution of Eqn. 1 would not only require adding more constraints, but also make the solution approach dependent on the surface representation. The approach to solving Eqn. 1 presented here consists in introducing a minimal set of four independent natural geodesic variations and computing the gradient of ε with respect to these variations explicitly. We introduce the following four natural variations per geodesic i: (1) the infinitesimal displacement $\mathrm{d}s_P^i$ of the geodesic's start point P^i in tangential direction; (2) the infinitesimal displacement $\mathrm{d}\beta_P^i$ of P^i in binormal direction; the infinitesimal clockwise rotation $\mathrm{d}\theta^i$ of the geodesic's initial direction; and (4) the infinitesimal length increment $\mathrm{d}\ell^i$ of the geodesic's length for a fixed point P^i. Let $\mathrm{d}\xi^i$ denote the four variations of geodesic γ^i, then it holds for the global path-error Jacobian

$$\mathbf{J} = \frac{\partial \varepsilon}{\partial \xi} \in \mathbb{R}^{4n\times 4n}, \quad \mathrm{d}\xi = \begin{bmatrix} \mathrm{d}\xi^1 & \dots & \mathrm{d}\xi^i & \dots & \mathrm{d}\xi^n \end{bmatrix} \in \mathbb{R}^{4n}.$$

Stating the path-error Jacobian explicitly requires information about how geodesics displace when the initial conditions are varied by $\mathrm{d}\xi$. While it is straight-forward to compute the geodesic variations according to $\mathrm{d}s^i_P$ and $\mathrm{d}\ell^i$, the computation of the geodesic variations according to $\mathrm{d}\beta^i_P$ and $\mathrm{d}\theta^i$ (Fig. 1) requires solving two scalar Jacobi equations, i.e., second order differential equations, along the geodesic.

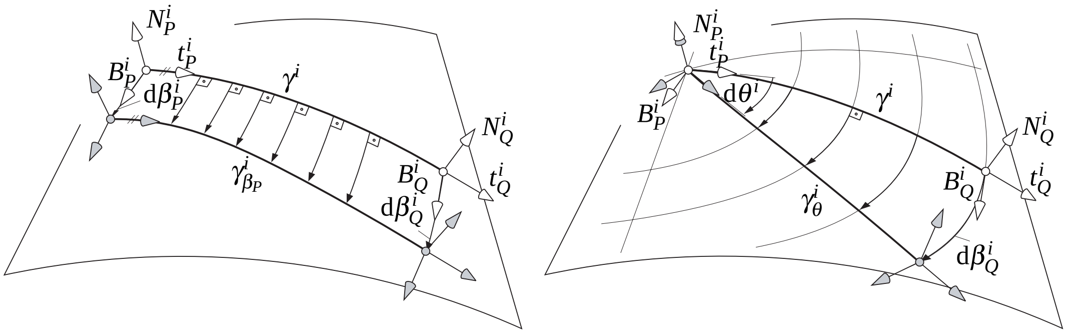

Figure 1: Left: An infinitesimal displacement $\mathrm{d}\beta^i_P$ of the geodesic's start point P^i in binormal direction causes a binormal displacement $\mathrm{d}\beta^i_Q$ of the end point Q^i. Right: Likewise, an infinitesimal clockwise rotation $\mathrm{d}\theta^i$ of the geodesic's initial direction causes a binormal displacement $\mathrm{d}\beta^i_Q$. Here, t^i is the geodesic's tangent, N^i is the surface's normal, and $B^i = t^i \times N^i$ is the respective binormal.

The explicit path-error Jacobian $\mathbf{J}$ allows for the efficient computation of finite parameter corrections $\Delta\xi = -\mathbf{J}\,\varepsilon$ which can be mapped back to finite corrections Δq of the geodesic parameters. $\mathbf{J}$ has band structure due to the local dependency of the path error on surface S^i from the geodesics γ^{i-1}, γ^i, and γ_{i+1}. Therefore, the method's computational costs grow linearly with the number of wrapping surfaces when using a band-matrix routine to solve for the geodesic corrections.
We applied our method in a dynamic simulation (Fig. 2a) in which a single muscle path wraps around four (nonsimple) surfaces: a cylinder, a torus, a parametric surface patch fitted to a human ribcage, and an elliptic torus. At one end, a freely moving point mass is attached to the muscle. With the proposed method, the exact path length and the exact rate of length change can be computed at any time step. We also evaluated the computational costs by wrapping a single muscle path over a variable number of sinusoidally (frequency $0.25\,\mathrm{rad/s}$) moving cylinders (Fig. 2b) using a desktop computer with an Intel i7, 3.5GHz. For 100, 500, and 1000 cylinders, the real-time factors were 0.12, 0.62 (both faster than real-time), and 1.34 (slower than real-time), respectively.

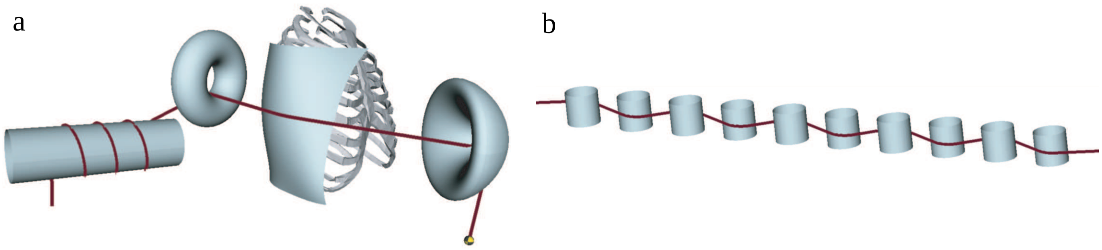

Figure 2: Wrapping over nonsimple surfaces (a) and over a variable number of cylinders (b)

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