

## Trajectory tracking problem for omnidirectional mobile robot with parameter variations and delayed feedback

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### Abstract

An omnidirectional mobile robot is a complex, controllable electromechanical system. The problem of tracking control of such type of wheeled mobile robots has attracted a lot of attention over the past twenty years [1-6].

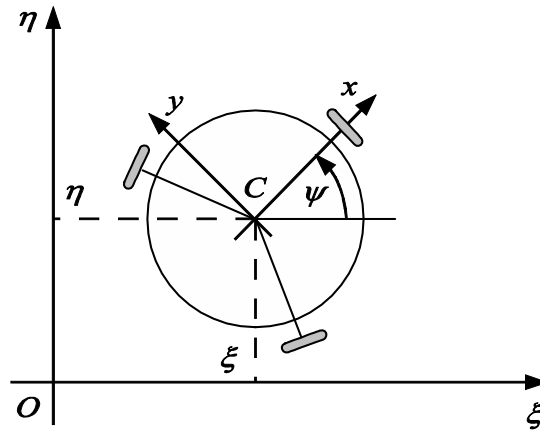


Figure 1: Model of the robot with three omnidirectional wheels.

The equations of dynamic model of the robot with three independent driving omnidirectional wheels equally spaced at 120 degrees from one another (fig. 1) are as follows [1,2].

$$\left\{ \begin{array}{l} (m + \Delta m)\ddot{\xi} + h\dot{\xi} + m_d\dot{\psi}\dot{\eta} = \sin\psi u_1 + \sin(\psi + \frac{2\pi}{3})u_2 + \sin(\psi + \frac{4\pi}{3})u_3 \\ (m + \Delta m)\ddot{\eta} + h\dot{\eta} - m_d\dot{\psi}\dot{\xi} = -\cos\psi u_1 - \cos(\psi + \frac{2\pi}{3})u_2 - \cos(\psi + \frac{4\pi}{3})u_3 \\ (I + \Delta I)\ddot{\psi} + 2a^2 h\dot{\psi} = -a(u_1 + u_2 + u_3) \end{array} \right. \quad (1)$$

Here  $\xi$  and  $\eta$  are coordinates of the center of a platform of the robot in Cartesian system  $O\xi\eta$ ;  $\psi$  is the platform angle of rotation round a vertical counted from an axis  $\xi$ ;  $u_1$ ,  $u_2$  and  $u_3$  are control tensions, given on electric motors;  $a$  is a distance from the center of a platform to the center of each wheel; the constant  $h$  is defined by coefficient of the moment of the antielectromotive force and radius of a wheel;  $m$ ,  $I$  and  $m_d$  are the known components of masso-inertial parameters of the system;  $\Delta m$  and  $\Delta I$  are the unknown components of mass of a platform and its moment of inertia respectively satisfying to restrictions:  $|\Delta m| < \Delta m_0 = \text{const}$ ,  $|\Delta I| < \Delta I_0 = \text{const}$ , and  $\Delta m_0 < m$ ,  $\Delta I_0 < I$ .

We assume that at structure of feedback there is some unknown variable delay  $\tau(t) < \tau_0 = \text{const} > 0$ , which arises owing to delays in operation of devices of a control system and final speed of signaling on a network [6]. Let  $\mathbf{q}_0(t) = (\xi_0(t), \eta_0(t), \psi_0(t))^T$  is some twice continuously differentiable limited function defined at all  $t > -\tau_0$ .

The problem of trajectory tracking of the robot consists in the following. It is required to find a control  $\mathbf{u}(t - \tau(t))$  ( $\mathbf{u} = (u_1, u_2, u_3)^T$ ,  $|\mathbf{u}| < u_0 = \text{const}$ ) and to specify the restrictions on parameters of the system and a trajectory, at which for some number  $\varepsilon > 0$  (tracking error) will be numbers  $\delta > 0$  and  $t^* > 0$ , such that for any initial function  $\varphi(s)$ ,  $-2\tau_0 < s < 0$ , satisfying to restriction

$$\max_{-2\tau_0 \leq s \leq 0} |(\varphi(s) - \mathbf{q}_0(s), \dot{\varphi}(s) - \dot{\mathbf{q}}_0(s))| < \delta$$

for all solutions of system (1) with condition  $\mathbf{q}(s) = \varphi(s)$ ,  $-2\tau_0 < s < 0$ , for all  $t > t^*$  the inequality  $|\mathbf{q}(t) - \mathbf{q}_0(t)| < \varepsilon$  will take place.

With use of the works [7,8] the control laws solving a problem of trajectory tracking of the robot taking into account both nonlinearity and non-stationarity of the system and unknown mass-inertial characteristics and delay are found.

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