Wave-Based Attitude Control of Spacecraft with Fuel Sloshing Dynamics

J. W. Thompson*, **W. J. O'Connor#**,

School of Mechanical and Materials Engineering
University College Dublin
Belfield, Dublin 4, Ireland
* joseph.thompson@ucdconnect.ie
# william.oconnor@ucd.ie

Abstract
Many mechanical systems are inherently flexible, making it difficult to achieve rapid, controlled motion. The control challenge is even greater when the system is not well modelled, has dynamics that change with time, or is under-actuated. A rocket with sloshing fluid propellant is an extreme case. Many control strategies struggle with such systems. However a wave-based control method has been shown to cope well with these challenges [1]. The key idea is that the motion of the actuator can be separated into two notional components, one travelling from the actuator into the system, the other leaving the system through the actuator. Intuitively the actuator simultaneously launches mechanical waves into a system while it absorbs returning waves. When the launching and absorbing is finished vibrations have been damped and the desired reference motion is left behind. The method has been demonstrated to work well for 1-D and 2-D lumped flexible systems and in robotic and crane applications. The aim of this paper is to extend the application to the control of spacecraft with flexible structures and appendages (e.g. solar panels), and with on-board liquid propellant. Such spacecraft may be modelled as multi rigid body systems. This new area of application presents many new challenges. The spacecraft systems are often nonlinear, their associated flexibility is non-uniform, the sloshing dynamics are difficult or impossible to predict, and sensors and actuators can behave far from the ideal. Figure 1(a) shows a model of a planar

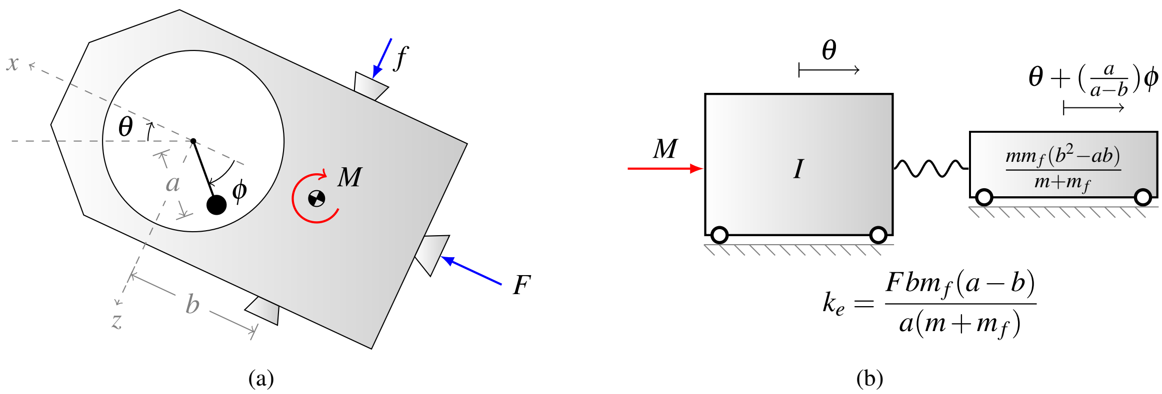

Figure 1: (a) 4-DOF planar rocket model, and (b) simplified 2-DOF attitude dynamics model.

upper-stage accelerating rocket consisting of a rigid rocket body of mass m and moment of inertia I, and a pendulum bob of mass m_f. The pendulum is connected to the body by a massless rod of length a, at a pivot point which is a length b from the body centre of mass. The pitch angle of the body is θ and the angle of the slosh pendulum w.r.t. the body is ϕ. F is the constant axial force providing the forward thrust to the rocket. f is a lateral force and M is an actuating moment. The nonlinear equations of motion of this system were simplified by assuming a constant axial acceleration a_x equal to the steady state value of

$$a_x = \frac{F}{m+m_f} \tag{1}$$

which corresponds to the equilibrium point $[\theta, \phi, \dot{\theta}, \dot{\phi}] = 0$. This assumption reduces the equations to a 2-DOF system in θ and ϕ. We assume only one control input is available, the pure moment M applied to the rocket body. The lateral force f is assumed to be zero. After linearization about this equilibrium point the equations may be written in the form:

$$I\ddot{\theta} = M + k_e(\phi_e - \theta) \tag{2}$$

$$I_e\ddot{\phi}_e = -k_e(\phi_e - \theta) \tag{3}$$

The system now resembles two inertias with an interconnecting spring. The first inertia is I and its displacement is θ and the values for the equivalent spring constant, second displacement and second inertia respectively, are calculated as:

$$k_e = \frac{Fbm_f(a-b)}{a(m+m_f)}, \quad \phi_e = \theta + \left(\frac{a}{a-b}\right)\phi, \quad I_e = \frac{mm_f(b^2-ab)}{m+m_f} \tag{4}$$

When the system is represented in this way the application of the force-impedance formulation of wave-based control [2] becomes more intuitive. The designed control law for pitch attitude control is:

$$M = k_0\left[\frac{1}{2}\theta_{ref} - \frac{1}{2}\left(\theta + \int_0^t \frac{M}{Z}dt\right)\right] \tag{5}$$

where θ_{ref} is the desired reference pitch angle, $Z = \sqrt{k_e I}$ is the wave impedance, k_0 is the proportional gain, and t is time. It was found that wave-based control can be successfully applied to the rocket system, albeit with some modifications. One of the advantages of wave-based control is that all measuring is done at the actuator, in this case the rocket body, so no measurement of the pendulum states is necessary, which is a significant bonus given the challenge of measuring or modelling them. When the ratio of inertias $\frac{I_e}{I}$ is much less than one, the effect of the pendulum on the body is much reduced and so it takes longer to fully suppress sloshing motions. This is acceptable, however, because in this case, by definition, the sloshing does not cause a major problem for the rocket controller. On the other hand, the control challenge is greatest when the fluid inertia ratio is large, and this is precisely when the new strategy delivers much improved performance. Further reduction of the sloshing settling time is possible by using an extra feedback from the lateral acceleration of the rocket. This extra feedback is analagous to measuring the force in the connecting spring in figure 1(b). This is in reality the moment being applied to the rocket body by the pendulum and can be estimated from the rocket lateral acceleration. Controllers were tested by numerical simulation. Suitable parameters for the model in figure 1 were chosen to represent the *ESC-A*, upper stage of the *Ariane 5* launcher. A single slosh pendulum was included to represent the primary sloshing mode for the rocket fuel tank when half full [3]. The ratio of inertias for this case is approximately 10. Sample results are shown in figure 2 for a 0.2 radian step change in θ_{ref}.

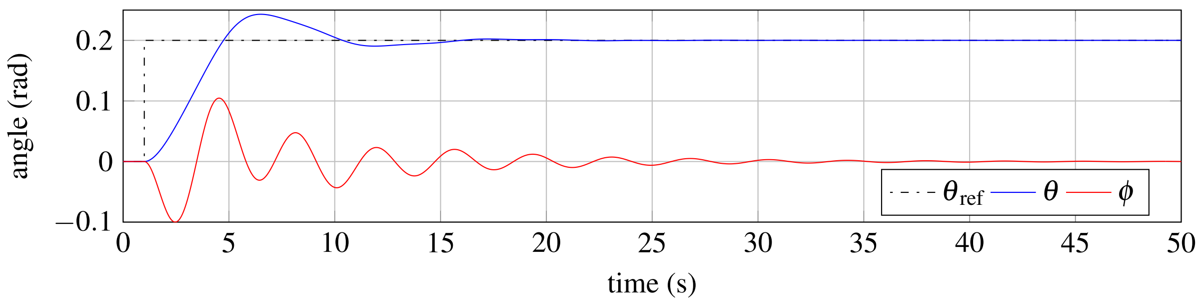

Figure 2: ESC-A pitch θ and pendulum angle ϕ for a 0.2 radian step change in θ_{ref}.

References

[1] W. J. O'Connor and D Lang. Position control of flexible robot arms using mechanical waves. ASME Journal of Dynamics Systems, Measurement and Control, 120(3):334–339, 1998.

[2] W. J. O'Connor. Excellent control of flexible systems via control of the actuator-system interface. European Control Conference, 2005.

[3] F. T. Dodge. The New Dynamic Behavior of Liquids in Moving Containers. Southwest Research Inst., San Antonio, TX, 2000, pp. 43 - 59.