

The Logarithmic Complexity Procedure for Parallel Multibody Dynamics Solution

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Abstract

The most effective methods for the parallel multibody dynamics solution are the ones with the logarithmic complexity usually based on the Divide and Conquer (DAC) algorithm [1, 2, 3]. The paper describes the new concept of a method with logarithmic complexity, with several advantages in comparison to the traditional DAC procedures. The method is based on the modified state space and the efficient set of natural coordinates [4]. Using the procedure described in [5] exploiting the Schur complement the resulting system of equations of motion (EOM) is obtained:

$$\mathbf{M}\dot{\mathbf{s}} + \mathbf{J}^T \boldsymbol{\mu} = \mathbf{p}^* \quad (1)$$

$$\mathbf{J}\dot{\mathbf{s}} = -\alpha \mathbf{f}(\mathbf{s}) \quad (2)$$

Where \mathbf{M} is the diagonal mass matrix, \mathbf{J} is the Jacobi matrix corresponding to the constraints \mathbf{f} , α is the coefficient of the Baumgarte stabilization, \mathbf{s} is the vector of natural coordinates describing the absolute system position, \mathbf{p}^* is the modified momentum of the system and $\boldsymbol{\mu}$ is the vector of the new Lagrange multipliers. Expressing $\dot{\mathbf{s}}$ from (1) and substituting into (2) the resulting system for unknown $\boldsymbol{\mu}$ is obtained:

$$\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T \boldsymbol{\mu} = \alpha \mathbf{f}(\mathbf{s}) + \mathbf{J}\mathbf{M}^{-1}\mathbf{p}^* \quad (3)$$

which can be simply written as follows:

$$\mathbf{A}\boldsymbol{\mu} = \mathbf{b} \quad (4)$$

The system of equations (4) is sparse, symmetric, positive definite with band structure for the case of a simple kinematic chain (Fig. 1 a). The system (4) has a structure of blocks (Fig. 1 b) corresponding to particular bodies with equivalent (small) sizes.

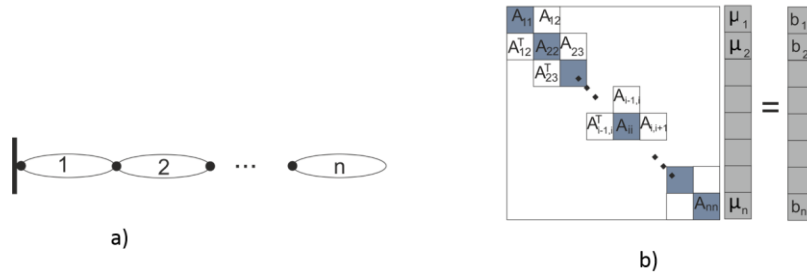


Figure 1 a): The simple kinematic chain b) The resulting matrix-structure

Thus the whole system of the equations can be understood as a set of the interconnected subsystems representing by the blocks for unknown vectors $\boldsymbol{\mu}_i$:

$$\mathbf{A}_{i-1,i}^T \boldsymbol{\mu}_{i-1} + \mathbf{A}_{ii} \boldsymbol{\mu}_i + \mathbf{A}_{i,i+1} \boldsymbol{\mu}_{i+1} = \mathbf{b}_i \quad (5)$$

The unknown multipliers $\boldsymbol{\mu}_i$ can be expressed from (5):

$$\boldsymbol{\mu}_i = \mathbf{A}_{ii}^{-1} (\mathbf{b}_i - \mathbf{A}_{i-1,i}^T \boldsymbol{\mu}_{i-1} - \mathbf{A}_{i,i+1} \boldsymbol{\mu}_{i+1}) \quad (6)$$

The indicated expression can be done for all odd unknown vectors $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\mu}_3, \dots$ and they can be substituted into the even subsystems. Thus the odd subsystems are eliminated from the even ones, the number of equations is reduced to a half and the structure of the dependence on the two neighbours is preserved. This procedure can be repeated and the size of the problem is reduced to a half in each level.

The resulting unknown is evaluated on the last level and the back-substitution follows with final evaluation of the particular unknowns μ_i . The process of the odd/even elimination can be done in parallel and the logarithmic complexity is obtained.

The results show, that if the number of bodies in the system is below some limit (approx. 1000), the direct solution of the original system is more effective using the Cholesky decomposition. That leads to an idea to combine the parallel elimination with the Cholesky decomposition. The eliminations is repeated as long as the limit system is achieved and it is then solved using the Cholesky decomposition. The Fig. 2 represents the results comparing the complete elimination (similar to the traditional DAC methods) with the modified method of the combination. The different limit system corresponds to L bodies involved.

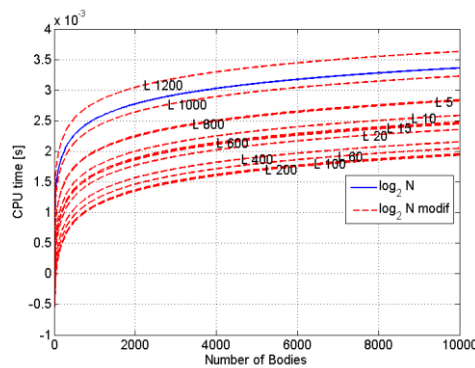


Figure 1: Comparison of the simple elimination procedure ($\log_2 N$) with the combination of the elimination and the Cholesky decomposition ($\log_2 N$ modif) for different limit (L).

In the case that less processors than the bodies are available, it is not effective to create the blocks corresponding to the particular bodies, but create as many blocks as the number of processors. The resulting complexity of the particular matrix operation slightly grows. However, the experiments show, it is more effective than to carry out the elimination with smaller sizes of the matrices, but partly in parallel and partly in sequential, which is the case of the traditional DAC algorithms. Another advantage of this method is the using of the natural (absolute) coordinates, it is thus not necessary to carry out the two initial phases for the position and the velocity evaluation as in the case of DAC with relative coordinates. Finally, this method enables the combination of the effective procedure of Cholesky decomposition with the parallel elimination process.

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