Structural Analysis, Regularization and Simulation for Multibody Systems of Index 3 and Larger

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Abstract

In the modeling and simulation of multi-body systems usually differential-algebraic equations (DAEs) in the form

$$\dot{p} = v \tag{1a}$$
$$M(p,t)\dot{v} = f(p,v,t) - G^T(p,t)\lambda - B^T(p,t)u, \tag{1b}$$
$$0 = g(p,t) \tag{1c}$$

arise, where the *unknown variables* are represented by the *position variables* p, the *velocity variables* v, and the *Lagrange multipliers* λ. Furthermore, a (possible) *control* is represented by the *control variables* u.

If the control u is known the differentiation index (d-index) of (1) is well known to be $\nu_d = 3$ if the so called Grübler condition is satisfied. In this case the model equations (1) contain so called *hidden constraints* up to level 2, in particular, the constraints on velocity level and on acceleration level which complicate the numerical treatment, e.g., instabilities, order reduction, convergence problems, or inconsistencies can occur in the numerical integration of (1).

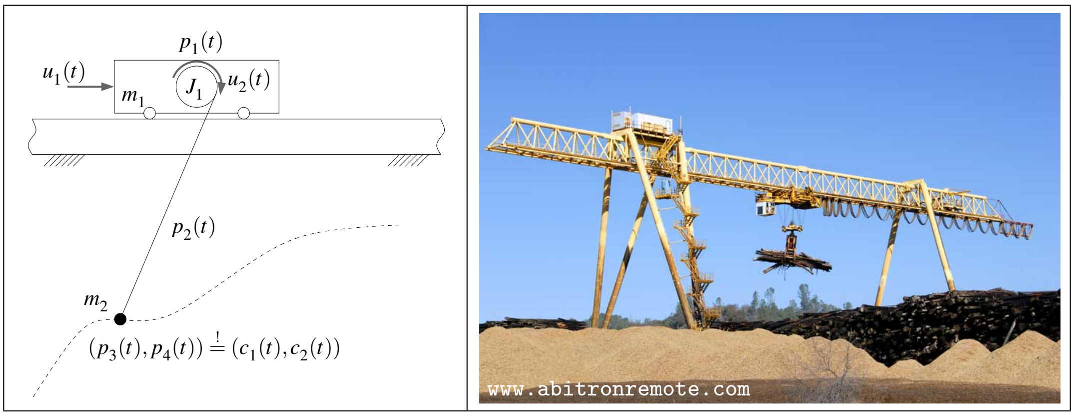

Figure 1: Bridge crane as an example for mechanical systems of high d-index (here d-index $\nu_d = 5$).

On the other hand, if we are interested in the determination of a control u forcing the dynamical system into a (partially) prescribed motion or to satisfy path constraints, i.e.,

$$0 = c(p(t),t), \tag{2}$$

then the resulting problem has the general form of a *quasi-linear DAE*

$$E(x,t)\dot{x} = f(x,t) \tag{3}$$

with $x^T = [p^T \; v^T \; \lambda^T \; u^T]$ as unknown variables. Those DAEs (3) are more general than (1) and covers model equations often used in industrial applications. In general, the index of (3) can increase arbitrarily, e.g., the d-index can be larger than 3. For example the model equations for the path control of a bridge crane, illustrated in Figure 1, form a quasi-linear DAE of d-index 5. Therefore, the DAE contains hidden constraints which are, roughly speaking, deeply hidden in the system. This deep hiddenness of constraints complicates the numerical integration extremely. Therefore, a regularization or remodeling of the model equations is required which

- *R1)* reduces the index and
- *R2)* preserves the set of solutions.

In modern simulation environments often a structural analysis based on the sparsity pattern of the system is used to obtain the required information on the hidden constraints. These hidden constraints can be added to the system equations and then, usually, dynamical state variables are selected for which the occurring derivatives are replaced by newly introduced algebraic variables (so-called dummy derivatives). Such a regularization may be valid only locally since the state selection can vary with the dynamical behavior of the system.

In this talk we will discuss the efficient and robust numerical integration of quasi-linear DAEs (3) of high index. We will propose an approach which combines a regularization with an efficient numerical integration.
The regularization approach uses the information obtained from the structural analysis to construct a regularized overdetermined system formulation which corresponds to a regularization satisfying R1) and R2) and is uniquely solvable if the original DAE was uniquely solvable. This overdetermined system can then be solved using specially adapted numerical integrators without the need of state selection and the introduction of new algebraic variables.
Based on that regularization approach we present the software package `QUALIDAES`. This software package is suited for the direct numerical integration of the proposed regularized formulation and uses adapted discretization methods based on the Runge-Kutta method of type RADAU IIa of order 5.
The efficiency and applicability of the proposed approach for the numerical treatment of quasi-linear DAEs (3) of high index will be demonstrated on several examples. Furthermore, a comparison to other widely used solvers like `RADAU5` and `DASSL/DASPK` will be provided.