ECCOMAS Thematic Conference on Multibody Dynamics
June 29 - July 2, 2015, Barcelona, Catalonia, Spain

Use of the weighting minimum norm solution for redundantly constrained multibody systems

María D. Gutiérrez-López [*], **José L. Bueno-López** [*], **Javier García de Jalón** [#]

[*] INSIA
Universidad Politécnica de Madrid
Carretera de Valencia km 7, 28031 Madrid, Spain
javier.garciadejalon@upm.es

[#] ETSII and INSIA
Universidad Politécnica de Madrid José
Gutiérrez Abascal 2, 28006 Madrid, Spain
javier.garciadejalon@upm.es

Abstract

It is very common to find papers in the literature where, after setting the dynamic equations, it is assumed that the constraint equations are independent and therefore the Jacobian matrix has full rank. However, in many situations it is possible to get dynamic equations with redundant constraints. One important example leading to the use of redundant constraints is the case of over-determined multibody systems, such as the numerous exceptions to the Grübler–Kutzbach criterion.

There has been a renewed interest in the determination of constraint forces in over-constrained multibody systems in the literature largely due to the work of Wojtyra and Frączek [1]. These constraint forces are influenced by a large variety of factors difficult to know or to estimate. Probably the most important factors are the following ones: joint flexibilities, link or body flexibilities, manufacturing errors regarding distances and angles. As some of these factors are difficult to know exactly, it is probably meaningless to talk about the "real solution". As real solutions are very difficult or unattainable, it is necessary to concentrate on "engineering solutions" that are sufficient for the design of real systems, which shall be approximate, cheap and safe. In this paper a simple mathematical solution for over-constrained multibody systems that was introduced for the first time in [2] is discussed in detail.

In an ideal system (with no friction) with redundant constraints, under some very easy to fulfill assumptions, the resultant constraint forces are:

$$\mathbf{\Phi}_{\mathbf{q}}^{T}\boldsymbol{\lambda} = \mathbf{F}\left(\mathbf{q},\dot{\mathbf{q}}\right) - \mathbf{M}\ddot{\mathbf{q}} \tag{1}$$

where $\mathbf{q}$ is the vector of Cartesian coordinates that defines the system position, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are its first and second order time derivatives, $\mathbf{M}$ is the inertia or mass matrix, $\mathbf{F}$ is a vector that includes the external and velocity dependent inertia forces, $\mathbf{\Phi}_{\mathbf{q}}$ is the Jacobian matrix of the kinematic constraint equations and $\boldsymbol{\lambda}$ the vector of Lagrange multipliers. These constraint forces are determined, although the individual Lagrange multipliers $\boldsymbol{\lambda}$ are not, because the rank of $\mathbf{\Phi}_{\mathbf{q}}^{T}$ is lower than its number of columns. Mathematically, system (1) has infinite solutions, but in Linear Algebra there is a solution that is considered as a preferred one: the minimum Euclidean norm solution. However, as was emphasized by Wojtyra and Frączek [1], the minimum norm solution of the linear system of equations (1) does not remain invariant under a change in units. Nevertheless, this issue can be addressed if the constraint equations meet some conditions, which can be met by multiplying them by a weighting matrix.

A possible way to eliminate redundancy is to introduce flexibility in a subset or in all bodies. The use of the weighted minimum norm solution can provide an easy alternative to the use of flexible bodies for the determination of redundant constraint forces. This purely numerical technique is similar to the relationship introduced by González and Kövecses [3] between penalty factors and physical stiffness. The stiffness of some bodies can be accounted for by multiplying the constraint equations by a set of weighting factors which depend on the stiffness properties. If the constraint equations are multiplied by a diagonal weighting matrix $\mathbf{W}$ whose elements are related to the stiffness distribution of the system, $\mathbf{W\Phi} = \mathbf{0}$, the constraint forces are obtained from the equations:

$$\left(\boldsymbol{\Phi}_{\mathbf{q}}^{T}\mathbf{W}\right)\boldsymbol{\lambda}=\mathbf{F}\left(\mathbf{q},\dot{\mathbf{q}}\right)-\mathbf{M}\ddot{\mathbf{q}} \tag{2}$$

The minimum norm solution of system (2) shall coincide with the solution with flexible behavior. The values of the diagonal matrix **W** depend on the Jacobian matrix of constraint equations $\boldsymbol{\Phi}_{\mathbf{q}}^{T}$ and on the stiffness properties of the system.

Unlike in [2], where this mathematical solution was presented for the first time through its application to two examples with only one set of self-equilibrating forces (one redundant constraint equation), this paper provides a further discussion of the physical meaning of this weighting minimum norm solution and gives a generalized method for obtaining the weighting matrix **W**. It is also applied to some more complex examples with a higher number of redundant constraint equations.

References

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