## Simultaneous estimation of kinematic state and unknown input forces in rigid-link multibody systems

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## Abstract

State observers are widely used to estimate unmeasured state variables in multibody systems (MBS) whenever it is necessary to synthesize state feedback controllers, virtual sensors or fault detection schemes. An observer is usually designed to reconstruct missing state variables from accurate system models, and the knowledge of the time histories of the measured inputs and outputs. Clearly, model uncertainty may seriously compromise the accuracy of observers and affect their practical usefulness. The typical approach to state estimation in MBS relies on Kalman filters based on dynamic models. On the one hand, dynamic models involve a large number of parameters (e.g. geometrical dimensions and inertial properties of the links, friction model), which increase the model uncertainty and accuracy. On the other, the measured input of the observer should comprise all the forces (and/or torques) exciting the system analyzed, which are often difficult to be measured. This issue does not allow implementing state observers, unless the unknown forces are treated as unknown system parameters [1] and therefore included in the augmented state together with positions and velocities, and roughly modelled by means of the so called random walk. Such an approach is also useful to estimate the unmeasured input forces, needed in several applications, e.g. for evaluating the interaction forces with the environment in manufacturing or medical devices. However, adopting such an approximate model, dictating that unknown forces should be represented through a constant value plus white noise (i.e. with infinite spectrum), may negatively affect the estimation of the kinematic state variables.

In the recent paper [2], it has been proved that the estimation of the kinematic variables in MBS, under the hypotheses of rigid links and negligible joint clearance, can be based on just the kinematic constraint equations and on kinematic measurements, through the so-called Kinematic Kalman Filter (KKF). Indeed, the use of kinematic equations imposes a lower number of parameters compared to dynamic ones, and hence an higher accuracy, and does not require any force measurement.

By exploiting such an approach, this paper proposes a novel technique for the simultaneous estimation of the kinematic state and the unknown inputs for MBS with rigid links and negligible joint clearance. The basic idea consists in splitting the estimation process into two observes running simultaneously: a KKF and a force observer. The KKF estimates positions, velocities and accelerations regardless of the knowledge of the external forces, and hence unbiased by the uncertainty introduced by any force approximate model. Its estimates are fed as the input of the force observer. Such an observer estimates both the measured and the unmeasured forces, respectively modelled by means of the inverse dynamic problem and the random walk. Besides belonging to the state, the known forces are employed as outputs of the force observer, in order to correct the estimation of the unknown input forces.

A schematic representation of the proposed approach is shown in Figure 1, by the kinematic scheme of the slider-crank mechanism used for numerical validation, and whose uncertain geometrical and inertial model parameters are collected in Table 1,. In particular, in the representation in Figure 1,  $k \in \mathbb{N}^+$  is the time step index, superscript "-" indicates the model-based a-priori estimates, superscript "+" denotes the a-posteriori estimates obtained updating (i.e. correcting) the a-priori estimates with the measurements. For the test it is supposed that the crank angular position and the slider acceleration are measured and used in the KKF to estimate the crank angular velocity and acceleration to grant

observability [2]. So as to meet the same requirement in the second observer, the torque driving the crank is also assumed to be measured, and the external force applied to the slider is estimated. Both the observers are based on the Extended Kalman Filter (EKF). Examples of the time histories of the measured signals are plotted in Figure 2.(a), (c) and (d). As a benchmark, the traditional EKF based on the dynamic model and a force random walk model is considered. The same set of measurements is employed in the proposed and benchmark observers. The results (see Figure 2 (b), (d) and (f)) and the comparison with the actual state clearly highlight the better performances ensured by the proposed approach, which always ensures smaller root-mean-square values of the estimation errors ( $e_{rms}$ ).

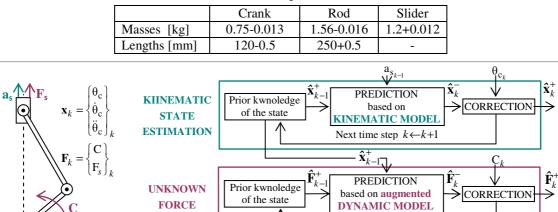
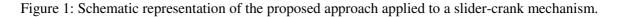


Table 1: Geometric and inertial parameters (nominal + error)



Next time step  $k \leftarrow k+1$ 

ESTIMATION

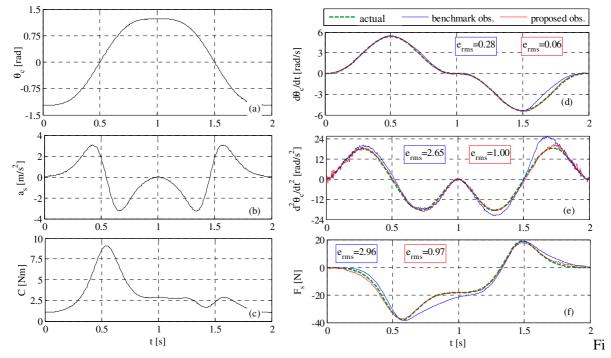


Figure 2: Measured variables: crank angle (a), slider acceleration (b), torque acting on the crank (c). Estimated variables: crank angular velocity (d) and acceleration (e), force acting on the slider (f).

## References

- [1] M. S. Grewal, A. P. Andrews. Kalman filtering: Theory and Practice Using MATLAB. 3rd Edition, Wiley, 2008.
- [2] I. Palomba, D. Richiedei, A. Trevisani. Nonlinear kinematic state estimation in rigid-link multibody systems by spherical simplex sigma point unscented Kalman filters. In Proceedings ISMA International Conference on Noise and Vibration Engineering, Leuven, Belgium, 2014.