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Approximative modeling of compliant mechanism dynamics

M. Roesner*, **R. Lammering***, **R. Friedrich***

* Institute of Mechanics, Department of Mechanical Engineering
Helmut-Schmidt-University/ University of the Federal Armed Forces Hamburg
Holstenhofweg 85, 22043 Hamburg, Germany
malte.roesner@hsu-hh.de

Abstract
In this work, the dynamic characteristics of compliant mechanisms consisting of flexure hinges are approximated by a novel approach based on the transformation of the compliant mechanism into an elastic multibody system. Instead of the entire mechanism only its significantly deformed domain are modeled by 3-D structural solids. Applying an appropriate model order reduction procedure to this elastic body, the flexure hinge may be represented by a small-dimensional stiffness and mass matrix being connected to point masses representing the rigid parts and therewith forming the compliant mechanism. This methodology results in highly accurate models with decisively less degrees of freedom. The proposed procedure is motivated by obvious drawbacks of conventional modeling techniques. With regard to control strategies, ultra-precise application and real time processing, large number of degrees of freedom or unacceptable deformation errors restrict their utilization.

1 Introduction

A new approach to develop a feed unit of ultra-precise machine tools for small workpieces is based on the application of compliant mechanisms (CM). This monolithic structure incorporates elastically deformable flexure hinges (FH) providing a relative motion between two adjacent structures of high stiffness, compare Figure 1(a). The elastic deformation is enabled by the local decrease of the bending stiffness via diminishment of the cross-section [1]. Currently, non-intuitive design and optimization techniques are in the main focus of research as well as modeling and controlling strategies [2].
A brief summary of drawbacks of common modeling approaches is given in Table 1, where TBT is the Timoshenko beam theory and PRBM relates to the pseudo-rigid-body model, to motivate the novel approach for dynamic modeling of CM stated in this work.

Table 1: Summary of advantages and disadvantages of different modeling approaches

Model	Advantages	Disadvantages
FEM	+ accurate results + easy to generate	- multitude of DOF - inapplicable for control
TBT	+ analytical model + easy to implement	- imprecise results - inapplicable for CMs
PRBM	+ few DOF + easy to implement	- imprecise results - inapplicable for CMs

The application of CM as feed unit demands a small system size of less than 100 degrees of freedom (DOF) and accurate approximation of dynamical performance to assure ultra-precise motion in real time control. The approximation of the CM as an elastic multibody system (EMBS) in combination with model order reduction (MOR) fulfills these requirements and offers a scheme of general applicability.

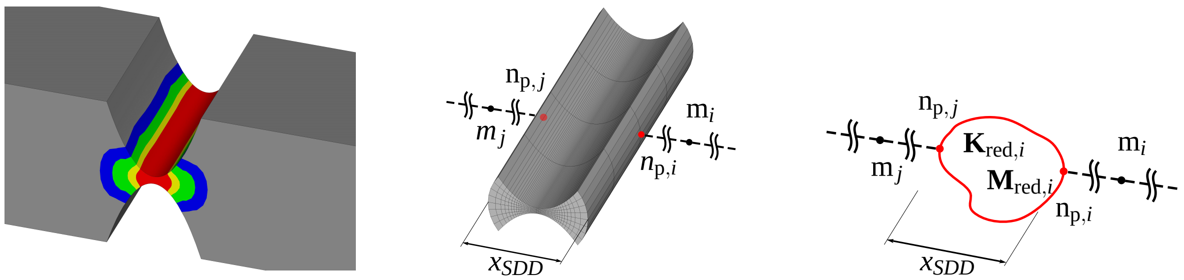
2 Reduced elastic multibody system representation of compliant mechanisms

The following three steps are involved to find a reduced elastic multibody system (REMBS) representation of CM.

First, a static finite element analysis of the FH incorporated in the CM is performed to determine the elastic body dimensions by means of the significantly deformed domain (SDD). Under the assumption of linear elasticity and isotropic material behaviour, nodes in the range of

$$t_{SDD} \leq \frac{\varepsilon^{el}_{eq,node}}{\varepsilon^{el}_{eq,max}} \leq 1 \tag{1}$$

are selected, where t_{SDD} is a user defined value, $\varepsilon^{el}_{eq,node}$ is the resulting equivalent elastic strain at each node and $\varepsilon^{el}_{eq,max}$ refers to the maximal occurring equivalent elastic strain. Figure 1(a) shows the selected domains for different values of t_{SDD}, where blue belongs to $t_{SDD} = 0.01$, green to $t_{SDD} = 0.02$, yellow to $t_{SDD} = 0.05$, and red to $t_{SDD} = 0.10$, respectively.
In a second step, the adjacent elements to the selected nodes form the SDD which is further treated as elastic body of length x_{SDD}. The remaining structure is assumed to be rigid and represented by point masses m_i with appropriate dynamical properties, as depicted in Figure 1(b). The point masses are rigidly connected to the elastic body via linking nodes $\mathrm{n}_{\mathrm{p},i}$ and $\mathrm{n}_{\mathrm{p},j}$ such that an elastic multibody system (EMBS) arises.
Since the elastic body consists of several thousand DOF, MOR using the component mode synthesis, as described in [3], is applied in the third step yielding a reduced stiffness $\mathbf{K}_{\mathrm{red}}$ and mass matrix $\mathbf{M}_{\mathrm{red}}$ of the elastic body. These matrices specify an arbitrary element with undefined geometry relating two nodes $\mathrm{n}_{\mathrm{p},i}$ and $\mathrm{n}_{\mathrm{p},j}$ with six DOF each, allowing easy connectivity, as shown in Figure 1(c). The REMBS of the CM is assembled by rigidly linking multiple point masses and pilot nodes appropriately.
This universally valid procedure enables highly accurate approximation of CM dynamics with considerable less DOF.



(a) Significantly deformed domain (SDD) defined by $\mathbf{K}_{\mathrm{full}}$ and $\mathbf{M}_{\mathrm{full}}$ for different values of t_{SDD}

(b) Elastic multibody system (EMBS) specified by $\mathbf{K}_{\mathrm{SDD}}$ and $\mathbf{M}_{\mathrm{SDD}}$

(c) Reduced elastic multibody system (REMBS) determined by $\mathbf{K}_{\mathrm{red}}$ and $\mathbf{M}_{\mathrm{red}}$

Figure 1: Steps to gain a reduced approximation of a flexure hinge

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References

[1] N. Lobontiu. Compliant mechanisms: Design of flexure hinges. CRC Press, 2010.

[2] J. Wulfsberg, R. Lammering, T. Schuster, N. Kong, M. Roesner, E. Bauma, R. Friedrich. A novel methodology for the development of compliant mechanisms with application to feed units. Production Engineering, Vol. 7, No. 5, pp. 503–510, 2013.

[3] J. Wijker. Spacecraft Structures. Springer Berlin Heidelberg, 2008.