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Synchronization-based state observer for impacting multibody systems using switched geometric unilateral constraints

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Abstract

We present a new design of a state observer for multibody systems subjected to unilateral constraints using only the information of the impact time. It extends the approach presented in [1] with a new concept of constraints called switched geometric unilateral constraints. These constraints introduce constraint forces in the kinematic equation which render the generalized coordinate discontinuous. The introduction of position jumps improves the synchronization rate and expands the applicability of the observer.

Dynamics

We consider a forced linear time-invariant multibody system subjected to unilateral constraints and position jumps. The dynamics is described by the measure differential inclusion (MDI)

$$\begin{pmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M} \end{pmatrix} \begin{pmatrix} \mathrm{d}\boldsymbol{q} \\ \mathrm{d}\boldsymbol{u} \end{pmatrix} = \begin{pmatrix} \boldsymbol{K}\boldsymbol{u} \\ -\boldsymbol{C}\boldsymbol{u} - \boldsymbol{K}\boldsymbol{q} + \boldsymbol{f}(t) \end{pmatrix} \mathrm{d}t + \boldsymbol{W} \begin{pmatrix} \boldsymbol{\sigma} \\ \boldsymbol{\lambda} \end{pmatrix} \mathrm{d}t + \boldsymbol{W} \begin{pmatrix} \boldsymbol{\Sigma} \\ \boldsymbol{\Lambda} \end{pmatrix} \mathrm{d}\eta. \tag{1}$$

Therefore, the generalized velocity $\boldsymbol{u}$ is generally no longer the derivative of the generalized coordinate $\boldsymbol{q}$. The differential measures $\mathrm{d}\boldsymbol{q}$ and $\mathrm{d}\boldsymbol{u}$ have a density w.r.t. to the Lebesgue measure $\mathrm{d}t$ and the atomic measure $\mathrm{d}\eta$.

The multibody system is subjected to *switched geometric unilateral constraints*. This new concept of constraints extends switched kinematic unilateral constraints (as introduced in [1]) with position jumps induced by *switched geometric bilateral constraints*. The constraint forces and constraint impulses are given by constitutive laws defined using the local kinematic quantities

$$\text{constraint distance:} \quad \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{W}^{\mathsf{T}}\boldsymbol{q} + \boldsymbol{q}_r, \tag{2}$$

$$\text{constraint velocity:} \quad \boldsymbol{\gamma}(\boldsymbol{u}) = \boldsymbol{W}^{\mathsf{T}}\boldsymbol{u} \neq \dot{\boldsymbol{g}}. \tag{3}$$

The switched kinematic unilateral constraints witch constraint force $\boldsymbol{\lambda}$ and impulse $\boldsymbol{\Lambda}$ are kinematic unilateral constraints (sprag clutches) which are switched on and off by an external Boolean switching function $\boldsymbol{\chi}(t) : \mathbb{R} \to \{0,\ 1\}^m$. The constraint force $\boldsymbol{\sigma}$ and the constraint impulse $\boldsymbol{\Sigma}$ of the switched geometric bilateral constraint are given by $g_i^+(\boldsymbol{q}^+) = 0$ for $\chi_i = 1$ and $\sigma_i = \Sigma_i = 0$ for $\chi_i = 0$ for each contact $i \in \{1, 2, \ldots, m\}$. These laws can be written as

$$-\sigma = \boldsymbol{L}_{\chi(t)}^{-1}\,\boldsymbol{\gamma}(\boldsymbol{u}), \tag{4}$$

$$-\Sigma = \boldsymbol{L}_{\chi(t)}^{-1}\,\boldsymbol{g}(\boldsymbol{q}^-), \tag{5}$$

where $\boldsymbol{L}_{\chi(t)}^{-1} \in \mathbb{R}^{m\times m}$ depends on $\boldsymbol{K}$, $\boldsymbol{W}$ and $\boldsymbol{\chi}(t)$.

Attractive incremental stability

The multibody system described by the MDI (1) subjected to switched geometric unilateral constraints is *attractively incrementally stable*. Therefore, any two solution curves $(\boldsymbol{q}_1(t),\ \boldsymbol{u}_1(t))$ and $(\boldsymbol{q}_2(t),\ \boldsymbol{u}_2(t))$ approach each other and remain close in the sense of Lyapunov.

A necessary condition for attractive incremental stability is the maximal monotonicity of the impact map, which is a natural assumption as shown in [2]. No assumptions are necessary for the switching function $\boldsymbol{\chi}(t)$. The proof in presented in the full paper.

Synchronization and observer design

The switched geometric unilateral constraints are equivalent to a geometric unilateral constraints if the switching function $\boldsymbol{\chi}(t)$ is given by the constraint distance $\boldsymbol{g}(t)$ as

$$\chi_i(t) = \begin{cases} 1 & \text{if } g_i(t) = 0, \\ 0 & \text{if } g_i(t) > 0, \end{cases} \quad \text{for } 1 \leq i \leq m. \tag{6}$$

With this insight we can use the property of attractive incremental stability to design a state observer using master-slave synchronization. The master and slave system are described by the MDI (1) subjected to switched geometric unilateral constraint. The switching function $\boldsymbol{\chi}(t)$ is defined by (6) using the constraint distance $\boldsymbol{g}(t)$ of the master system. Therefore, the (real) master system is subjected to geometric unilateral constraints, whereas the (artificial) slave system is a perfect replica with switched geometric unilateral constraints that are switched on when the corresponding contacts of the master system are closed.
The results are illustrated using simulations of the master-slave system as depicted in Figure 1. The evolution of Lyapunov function capturing the synchronization error is shown in Figure 2 for the case of switched kinematic (gray) and switched geometric (black) unilateral constraints. Synchronization is achieved in both cases, but the extension with position jumps improves the synchronization rate.

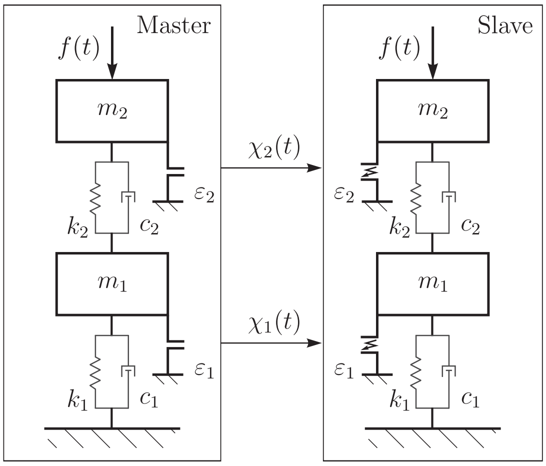

Figure 1: Master-slave system coupled by Boolean switching function $\boldsymbol{\chi}(t)$.

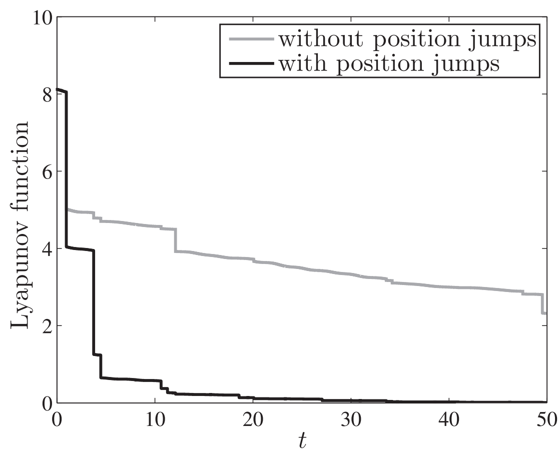

Figure 2: Lyapunov function without position jumps (gray) and with position jumps (black).

Conclusions

The slave system reproduces the full state of the master system using only the information of the impact time instants. The introduction of switched geometric unilateral constraints improves the synchronization rate and gives rise to the extension of this approach to systems with a rigid body mode.

Acknowledgement

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References

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